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Jury Shevchenko and Elena Skrydlova

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REDUCTION OF THE CENTROPROJECTIVE CONNECTION OF THE PROJECTIVE GROUP TO THE FUNDAMENTAL-GROUP CONNECTION OF A SURFACE

JURY SHEVCHENKO AND ELENA SKRYDLOVA

Abstract. The projective group effectively acting in the multi-measured projective space is represented by the principal bundle of the centroprojective frames, in which a symmetrical centroprojective connection is defined. A surface is considered in the projective space. The principal bundle above the surface with the typical fiber — a subgroup of the stationarity for the centered tangent plane to the surface in a fixed point is considered. Fundamental-group connection is given in this fibering. It consists of tangent affine and centroprojective, normal linear and affine-group connections. It is shown that the centroprojective connection in the projective group is reduced to the fundamental-group connection in the principal bundle associated with the surface in the projective space.

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1. CENTROPROJECTIVE CONNECTION IN THE PROJECTIVE GROUP

In n-dimensional projective space P_n, let us consider the moving frame $\{A, A_I\}$ $(I, \dots = \overline{1,n})$ with the derivation formulas

$$dA = \vartheta A + \theta^I A_I, \quad dA_I = \vartheta A_I + \theta_I^J A_J + \theta_I A, \tag{1.1}$$

where the form ϑ plays the role of a proportionality factor and the structure forms $\theta^I, \theta^I_J, \theta_I$ of the projective group GP(n), which acts effectively in the space P_n, satisfy the Cartan equations (see, e. g., [3])

$$D\theta^I = \theta^J \wedge \theta^I_J, \tag{1.2$_1$}$$

$$D\theta^I_J = \theta^K_J \wedge \theta^I_K + \theta^K \wedge \theta^I_{JK}, \tag{1.2$_2$}$$

$$D\theta_I = \theta^J_I \wedge \theta_J; \tag{1.2$_3$}$$

$$\theta^I_{JK} = -\delta^I_J \theta_K - \delta^I_K \theta_J. \tag{1.3}$$

The equations (1.2) are the structure equations of the principal fiber bundle of the centroprojective frames $C_{n(n+1)}(P_n)$, whose base is the projective space P_n (a region circumscribed by a point A). The centroprojective (coaffine) group $C_{n(n+1)} =$

$GA^*(n) \subset \mathrm{GP}(n)$ is the typical fiber. This group acts in any centroprojective space P_n^0 that is the space P_n with the fixed point A.

Proposition 1. *The fibering $C_{n(n+1)}(P_n)$ of the centroprojective frames has the principal quotient bundle $L_{n^2}(P_n)$ with the structure equations* (1.2_1), (1.2_2) *the same base P_n, and typical fiber $L_{n^2} = \mathrm{GL}(n)$, where $\mathrm{GL}(n)$ is the linear quotient group acting [2] ineffectively in the pencil of lines passing through the point A of the projective space P_n (this group acts in the projective quotient space $P_{n-1} = P_n/A$ (see, e. g., [1])).*

We give a centroprojective connection on the principal fiber bundle $C_{n(n+1)}(P_n)$ using Laptev–Lumiste method [4, 6, 8] by the forms

$$\hat{\theta}^I_J = \theta^I_J - \Pi^I_{JK}\theta^K, \quad \hat{\theta}_I = \theta_I - \Pi_{IJ}\theta^J, \tag{1.4}$$

where the components of the centroprojective connection object $\Pi = \{\Pi^I_{JK}, \Pi_{IJ}\}$ satisfy the differential equations

$$\Delta\Pi^I_{JK} + \theta^I_{JK} = \Pi^I_{JKL}\theta^L, \tag{1.5$_1$}$$

$$\Delta\Pi_{IJ} + \Pi^K_{IJ}\theta_K = \Pi_{IJK}\theta^K. \tag{1.5$_2$}$$

The tensor operator Δ acts by

$$\Delta\Pi^I_{JK} = d\Pi^I_{JK} + \Pi^L_{JK}\theta^I_L - \Pi^I_{LK}\theta^L_J - \Pi^I_{JL}\theta^L_K.$$

Proposition 2. *The centroprojective connection object Π is a quasitensor containing the quasitensor Π^I_{JK} which determines an affine (special linear) connection on the fibering $L_{n^2}(P_n)$ of linear frames.*

The forms of centroprojective connection (1.4) satisfy the structure equations

$$D\hat{\theta}^I_J = \hat{\theta}^K_J \wedge \hat{\theta}^I_K + K^I_{JKL}\theta^K \wedge \theta^L, \tag{1.6$_1$}$$

$$D\hat{\theta}_I = \hat{\theta}^J_I \wedge \hat{\theta}_J + K_{IJK}\theta^J \wedge \theta^K. \tag{1.6$_2$}$$

The components of the centroprojective curvature object $K = \{K^I_{JKL}, K_{IJK}\}$ are defined by

$$K^I_{JKL} = \Pi^I_{J[KL]} - \Pi^M_{J[K}\Pi^I_{ML]}, \quad K_{IJK} = \Pi_{I[JK]} - \Pi^L_{I[J}\Pi_{LK]}.$$

The square brackets are alternation in the extreme indices in these brackets. These components satisfy the differential comparisons modulo basis forms θ^I of the space P_n:

$$\Delta K^I_{JKL} \cong 0, \quad \Delta K_{IJK} + K^L_{IJK}\theta_L \cong 0.$$

Proposition 3. *The centroprojective curvature object K is a tensor containing the subtensor of the affine curvature K^I_{JKL}.*

We put the affine connection forms (1.4_1) into the structure equations (1.2_1) of the base P_n

$$D\theta^I = \theta^J \wedge \hat{\theta}^I_J + S^I_{JK}\theta^J \wedge \theta^K, \tag{1.7}$$

where $S^I_{JK} = \Pi^I_{[JK]}$ are the components of the affine torsion object of the centroprojective connection. The differential equations (1.5_1) with the symmetry of the forms θ^I_{JK} (1.3) in the lower indices imply the comparisons $\Delta S^I_{JK} \cong 0$.

Proposition 4. *The bundle $C_{n(n+1)}(P_n)$ of centroprojective frames with given centroprojective connection is the space $C_{n(n+1),n}$ of the centroprojective connection with closed structure equations* (1.6), (1.7) *which contain the affine torsion tensor S^I_{JK} and the centroprojective curvature object K. The space $C_{n(n+1),n}$ has the quotient space of an affine connection $L_{n^2,n}$* (1.7), (1.6_1) *with the torsion S^I_{JK} and curvature K^I_{JKL} tensors.*

According to [5], we can introduce the torsion object (centroprojective torsion object) of the centroprojective connection $S = \{S^I_{JK}, S_{IJ}\}$, where $S_{IJ} = \Pi_{[IJ]}$. The differential equations (1.5_2) imply

$$\Delta S_{IJ} + S^K_{IJ}\theta_K \cong 0.$$

Proposition 5. *The centroprojective torsion object S is a tensor containing the affine torsion subtensor S^I_{JK}.*

Definition. The centroprojective connection is an affine symmetric connection (an affine torsion-free connection) if $S^I_{JK} = 0$. The centroprojective connection is a symmetric connection (a centroprojective torsion-free connection) if $S = 0$.

Conclusion 1. *The forms θ^I_{JK}* (1.3) *in the differential equations* (1.5_1) *for the components of the affine subconnection object Π^I_{JK} are symmetric forms. Therefore, the affine connection have to be symmetric connection ($\Pi^I_{[JK]} = 0$). There are the symmetric components Π^I_{JK} in the differential equations* (1.5_2) *for the components Π_{IJ} of the object Π hence Π_{IJ} are symmetric components ($\Pi_{[IJ]} = 0$.) Therefore, we can put only symmetric centroprojective connection into the projective group* $\mathrm{GP}(n) = C_{n(n+1)}(P_n)$.

2. Fundamental-group connection associated with a surface

In the projective space P_n we shall consider m-dimensional surface S_m $(1 \le m < n)$ as the family of the centered tangent planes T_m. Let us partition the indices set as:

$$I = (i, a); \quad i, \dots = \overline{1, m}; \quad a, \dots = \overline{m+1, n}.$$

Let us put the tops A, A_i of the moving frame $\{A, A_I\}$ on the tangent plane T_m so that the top A coincides with the tangent point. According to (1.1), let us write the equations of the surface S_m in the form

$$\theta^a = 0, \tag{2.1$_1$}$$

$$\theta_i^a = \Lambda_{ij}^a \theta^j. \tag{2.1$_2$}$$

Closing the equations (2.1$_1$) we obtain $\Lambda^a_{[ij]} = 0$. Prolonging (2.1$_2$) we have

$$\Delta\Lambda_{ij}^a \approxeq 0, \tag{2.2$_1$}$$

$$\Delta\Lambda_{ij}^a = \partial\Lambda_{ij}^a + \Lambda_{ij}^b \omega_b^a - \Lambda_{kj}^a \omega_i^k - \Lambda_{ik}^a \omega_j^k, \tag{2.2$_2$}$$

where $\partial = d|_{S_m}$, $\omega = \theta|_{S_m}$, the symbol $\approxeq$ is the comparison modulo basis forms θ^i of the surface S_m.

Eliminating the principal forms θ^i, θ^a, θ_i^a of the equations (2.1) for the surface S_m from the structure forms θ^I, θ_J^I, θ_I of the projective group GP(n) we keep the secondary forms. They are called the fibre forms on the surface S_m. The basis forms θ^i and fiber forms ω_j^i, ω_i, ω_b^a, ω_a^i, ω_a satisfy the structure equations [1,9]

$$D\theta^i = \theta^j \wedge \omega_j^i; \tag{2.3}$$

$$D\omega_j^i = \omega_j^k \wedge \omega_k^i + \theta^k \wedge \omega_{jk}^i, \tag{2.4$_1$}$$

$$\omega_{jk}^i = \Lambda_{jk}^a \omega_a^i - \delta_j^i \omega_k - \delta_k^i \omega_j; \tag{2.4$_2$}$$

$$D\omega_i = \omega_i^j \wedge \omega_j + \theta^i \wedge \omega_{ij}, \tag{2.5$_1$}$$

$$\omega_{ij} = \Lambda_{ij}^a \omega_a; \tag{2.5$_2$}$$

$$D\omega_b^a = \omega_b^c \wedge \omega_c^a + \theta^i \wedge \omega_{bi}^a, \quad \omega_{bi}^a = -\Lambda_{ij}^a \omega_b^j - \delta_b^a \omega_i; \tag{2.6}$$

$$D\omega_a^i = \omega_a^j \wedge \omega_j^i + \omega_a^b \wedge \omega_b^i + \theta^j \wedge \omega_{aj}^i, \tag{2.7$_1$}$$

$$\omega_{aj}^i = -\delta_j^i \omega_a; \tag{2.7$_2$}$$

$$D\omega_a = \omega_a^i \wedge \omega_i + \omega_a^b \wedge \omega_b. \tag{2.8}$$

We obtain the structure equations (2.3)–(2.8) of the principal bundle $G_r(S_m)$ associated with S_m. The surface S_m is a base of the principal bundle $G_r(S_m)$. The subgroup of stationarity $G_r \subset$ GP(n) of the centered tangent plane T_m is a typical fiber. We have

$$r = \dim G_r = n(n+1) - m(n-m).$$

Associated fibering $G_r(S_m)$ has 4 simple quotient principal bundles [10] over the base S_m with the structure equations:

(1) (2.3), (2.4) — the tangent linear frame fibering $L_{m^2}(S_m)$ with the typical fiber L_{m^2} = GL(m), where GL(m) is linear quotient group acting ineffectively on the bunch of tangent lines passing through the center A (on the quotient space $T_{m-1} = T_m/A$);

(2) (2.3)–(2.5) — the tangent centroprojective frame fibering $C_{m(m+1)}(S_m)$ with the typical fiber $C_{m(m+1)} = GA^*(m)$, where $GA^*(m)$ is centroprojective (coaffine) quotient group acting effectively on the centered tangent plane T_m;
(3) (2.3), (2.6) — the normal linear frame fibering $L_{(n-m)^2}(S_m)$ with the typical fiber $L_{(n-m)^2} = \mathrm{GL}(n-m)$, where $\mathrm{GL}(n-m)$ is linear quotient group acting ineffectively in the $(n-m-1)$-dimensional projective space $P_{n-m-1} = P_n/T_m$ (see, e. g., [1]);
(4) (2.3), (2.4), (2.6), (2.7) — the fibering $H_{m^2-mn+n^2}(S_m)$ with the typical fiber $H_{m^2-mn+n^2}$. This fiber is:
   (a) stationarity subgroup for the centered tangent plane to m-dimensional surface of n-dimensional affine space A_n,
   (b) stationarity subgroup for the tangent straight lines subbunch on the straight line bunch of the space P_n with the center A,
   (c) stationarity subgroup for the quotient plane $T_{m-1} = T_m/A$ in P_n/A,
   (d) an affine quotient group [11] of the projective subgroup G_r.

According to the Laptev–Lumiste method, a connection in the principal bundle $G_r(S_m)$ is defined by the forms

$$\tilde{\omega}^i_j = \omega^i_j - \Gamma^i_{jk}\theta^k, \quad \tilde{\omega}_i = \omega_i - \Gamma_{ij}\theta^j,$$
$$\tilde{\omega}^a_b = \omega^a_b - \Gamma^a_{bi}\theta^i, \quad \tilde{\omega}^i_a = \omega^i_a - \Gamma^i_{aj}\theta^j, \quad \tilde{\omega}_a = \omega_a - \Gamma_{ai}\theta^i, \tag{2.9}$$

and the components of the fundamental-group connection object

$$\Gamma = \{\Gamma^i_{jk}, \Gamma_{ij}, \Gamma^a_{bi}, \Gamma^i_{aj}, \Gamma_{ai}\}$$

satisfy the differential equations [9]

$$\Delta\Gamma^i_{jk} + \omega^i_{jk} = \Gamma^i_{jkl}\theta^l, \tag{2.10$_1$}$$
$$\Delta\Gamma_{ij} + \Gamma^k_{ij}\omega_k + \omega_{ij} = \Gamma_{ijk}\theta^k, \tag{2.10$_2$}$$
$$\Delta\Gamma^a_{bi} + \omega^a_{bi} = \Gamma^a_{bij}\theta^j, \tag{2.10$_3$}$$
$$\Delta\Gamma^i_{aj} + \Gamma^b_{aj}\omega^i_b - \Gamma^i_{kj}\omega^k_a + \omega^i_{aj} = \Gamma^i_{ajk}\theta^k, \tag{2.10$_4$}$$
$$\Delta\Gamma_{ai} + \Gamma^j_{ai}\omega_j + \Gamma^b_{ai}\omega_b - \Gamma_{ji}\omega^j_a = \Gamma_{aij}\theta^j. \tag{2.10$_5$}$$

The connection object Γ has 4 simple subobjects [10]: Γ^i_{jk} — the tangent affine connection object, $\{\Gamma^i_{jk}, \Gamma_{ij}\}$ — the tangent centroprojective connection object, Γ^a_{bi} — the normal linear connection object, $\{\Gamma^i_{jk}, \Gamma^a_{bi}, \Gamma^i_{aj}\}$ — affine-group connection object. These subobjects give the fundamental-group connections in the corresponding quotient fiberings of the associated fibering $G_r(S_m)$.

Conclusion 2. *The forms* ω^i_{jk} (2.4$_2$), ω_{ij} (2.5$_2$) *from the differential equations* (2.10$_1$) *and* (2.10$_2$) *for the components of the centroprojective connection object*

$\{\Gamma^i_{jk}, \Gamma_{ij}\}$ are symmetric forms. Therefore, on the surface S_m we can consider only connection centroprojective torsion-free: $\Gamma^i_{[jk]} = 0$, $\Gamma_{[ij]} = 0$.

The fundamental-group connection forms (2.9) satisfy the structure equations

$$D\tilde{\omega}^i_j = \tilde{\omega}^k_j \wedge \tilde{\omega}^i_k + R^i_{jkl}\theta^k \wedge \theta^l, \quad D\tilde{\omega}_i = \tilde{\omega}^j_i \wedge \tilde{\omega}_j + R_{ijk}\theta^j \wedge \theta^k,$$
$$D\tilde{\omega}^a_b = \tilde{\omega}^c_b \wedge \tilde{\omega}^a_c + R^a_{bij}\theta^i \wedge \theta^j, \quad D\tilde{\omega}^i_a = \tilde{\omega}^j_a \wedge \tilde{\omega}^i_j + \tilde{\omega}^b_a \wedge \tilde{\omega}^i_b + R^i_{ajk}\theta^j \wedge \theta^k,$$
$$D\tilde{\omega}_a = \tilde{\omega}^i_a \wedge \tilde{\omega}_i + \tilde{\omega}^b_a \wedge \tilde{\omega}_b + R_{aij}\theta^i \wedge \theta^j,$$

where the components of curvature object

$$R = \{R^i_{jkl}, R_{ijk}, R^a_{bij}, R^i_{ajk}, R_{aij}\}$$

of the fundamental-group connections Γ are expressed by the formulas

$$R^i_{jkl} = \Gamma^i_{j[kl]} - \Gamma^m_{j[k}\Gamma^i_{ml]}, \quad R_{ijk} = \Gamma_{i[jk]} - \Gamma^l_{i[j}\Gamma_{lk]}, \quad R^a_{bij} = \Gamma^a_{b[ij]} - \Gamma^c_{b[i}\Gamma^a_{cj]},$$
$$R^i_{ajk} = \Gamma^i_{a[jk]} - \Gamma^l_{a[j}\Gamma^i_{lk]} - \Gamma^b_{a[j}\Gamma^i_{bk]}, \quad R_{aij} = \Gamma_{a[ij]} - \Gamma^k_{a[i}\Gamma_{kj]} - \Gamma^b_{a[i}\Gamma_{bj]}.$$

These components satisfy the differential comparisons [10]

$$\Delta R^i_{jkl} \approxeq 0, \quad \Delta R_{ijk} + R^l_{ijk}\omega_l \approxeq 0, \quad \Delta R^a_{bij} \approxeq 0,$$
$$\Delta R^i_{ajk} - R^i_{ljk}\omega^l_a + R^b_{ajk}\omega^i_b \approxeq 0, \quad \Delta R_{aij} + R^k_{aij}\omega_k + R^b_{aij}\omega_b - R_{kij}\omega^k_a \approxeq 0.$$

Theorem 1. *The curvature object R of the fundamental-group connection Γ is a tensor containing:*

(1) the curvature tensor R^i_{jkl} of the tangent affine connection Γ^i_{jk},
(2) the curvature tensor $\{R^i_{jkl}, R_{ijk}\}$ of the tangent centroprojective connection $\{\Gamma^i_{jk}, \Gamma_{ij}\}$,
(3) the curvature tensor R^a_{bij} of the normal linear connection Γ^a_{bi},
(4) the curvature tensor $\{R^i_{jkl}, R^a_{bij}, R^i_{ajk}\}$ of the affine-group connection $\{\Gamma^i_{jk}, \Gamma^a_{bi}, \Gamma^i_{aj}\}$.

3. Reduction of the centroprojective connection

We define a symmetric centroprojective connection object $\Pi = \{\Pi^I_{JK}, \Pi_{IJ}\}$. By the partition of each index into two indices the object Π will consist from the following essential components

$$\Pi = \{\Pi^i_{jk}, \Pi^a_{ij}, \Pi^i_{aj}, \Pi^a_{bi}, \Pi^i_{ab}, \Pi^a_{bc}, \Pi_{ij}, \Pi_{ai}, \Pi_{ab}\}.$$

In the differential equations (1.5) for the components of the object Π we expand the action of the operator Δ and partition the indices. We take the equations (2.1) of the surface S_m, apply the operator Δ to the subobjects and write result in the comparisons form for the components analogical to the components of the fundamental object Λ^a_{ij} and connection object Γ:

$$\Delta\bar{\Pi}^a_{ij} \approxeq 0, \quad (3.1_1)$$

$$\Delta\bar{\Pi}^i_{jk} + \bar{\Pi}^a_{jk}\omega^i_a - \delta^i_j\omega_k - \delta^i_k\omega_j \approxeq 0, \quad (3.1_2)$$

$$\Delta\bar{\Pi}^a_{bi} - \bar{\Pi}^a_{ji}\omega^j_b - \delta^a_b\omega_i \approxeq 0, \quad (3.1_3)$$

$$\Delta\bar{\Pi}^i_{aj} + \bar{\Pi}^b_{aj}\omega^i_b - \bar{\Pi}^i_{kj}\omega^k_a - \delta^i_j\omega_a \approxeq 0, \quad (3.1_4)$$

$$\Delta\bar{\Pi}_{ij} + \bar{\Pi}^k_{ij}\omega_k + \bar{\Pi}^a_{ij}\omega_a \approxeq 0, \quad (3.1_5)$$

$$\Delta\bar{\Pi}_{ai} - \bar{\Pi}_{ji}\omega^j_a + \bar{\Pi}^j_{ai}\omega_j + \bar{\Pi}^b_{ai}\omega_b \approxeq 0. \quad (3.1_6)$$

In (3.1) $\bar{\Pi} = \Pi|_{S_m}$. From the coincidence of the differential comparisons (2.2_1) and (3.1_1) we have

Lemma 1. *The subobject Π^a_{ij} (of the centroprojective connection object Π without the torsion) restricted to the surface S_m is identified with the fundamental object Λ^a_{ij}:*

$$\bar{\Pi}^a_{ij} = \Lambda^a_{ij}. \quad (3.2)$$

Comparing the differential equations (2.10) with the forms (2.4_2)–(2.7_2) and the comparisons (3.1_2)–(3.1_6) by means of Lemma 1 we obtain the equalities

$$\Gamma^i_{jk} = \bar{\Pi}^i_{jk}, \quad \Gamma_{ij} = \bar{\Pi}_{ij}, \quad \Gamma^a_{bi} = \bar{\Pi}^a_{bi}, \quad \Gamma^i_{aj} = \bar{\Pi}^i_{aj}, \quad \Gamma_{ai} = \bar{\Pi}_{ai}.$$

Theorem 2. *The symmetric centroprojective connection on the projective group* GP(n) *effectively acting in projective space P_n is reduced to the fundamental-group connection on the surface S_m of the space P_n.*

Remark 1. The components $\Pi^i_{ab}, \Pi^a_{bc}, \Pi_{ab}$ of the connection object Π are not used (compare [7, 12]) for the reduction of the object Π to the object Γ.

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Authors' addresses

Jury Shevchenko
Immanuel Kant Baltic Federal University, Department of Fundamental Mathematics, 14 A. Nevsky St., 236041 Kaliningrad, Russia

Elena Skrydlova
Immanuel Kant Baltic Federal University, Department of Computer Safety, 14 A. Nevsky St., 236041 Kaliningrad, Russia
E-mail address: `ESkrydlova@kantiana.ru`