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ON SYMMETRIZATION OF HIGHER ORDER JETS

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Abstract. We discuss geometric constructions transforming r-th order semiholonomic or nonholonomic jets into holonomic ones.

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Denote by $\mathcal{M}f$ the category of smooth manifolds and all smooth maps, by $\mathcal{M}f_m$ the subcategory of m-dimensional manifolds and their local diffeomorphisms and by $\mathcal{FM}_m$ the category of fibered manifolds with m-dimensional bases and fibered maps over local diffeomorphisms. Roughly speaking, the construction of higher order jets can be interpreted as a bundle functor on the category $\mathcal{FM}_m$ or on the product category $\mathcal{M}f_m \times \mathcal{M}f$.

First, the r-th *holonomic prolongation* J^rY of a fibered manifold $Y \to M$ is defined as the space of all r-jets of local sections of Y. Then J^r is a bundle functor on $\mathcal{FM}_m$ transforming a fibered manifold $Y \to M$ into its r-jet prolongation J^rY (with the projection onto Y) and any $\mathcal{FM}_m$-morphism $\varphi : Y_1 \to Y_2$ covering $\underline{\varphi} : M_1 \to M_2$ into $J^r\varphi : J^rY_1 \to J^rY_2$, $J^r\varphi(j^r_x\sigma) = j^r_{\underline{\varphi}(x)}(\varphi \circ \sigma \circ \underline{\varphi}^{-1})$.

Second, for every two manifolds M and N, the *bundle of holonomic jets* $J^r(M, N)$ is the space of all r-jets of M into N. Given a local diffeomorphism $f : M_1 \to M_2$ and a map $g : N_1 \to N_2$, we have the induced map $J^r(f, g) : J^r(M_1, N_1) \to J^r(M_2, N_2)$ defined by $J^r(f, g)(X) = (j^r_y g) \circ X \circ (j^r_x f)^{-1}$, where x is the source and y is the target of $X \in J^r(M_1, N_1)$. Then J^r is a bundle functor defined on the product category $\mathcal{M}f_m \times \mathcal{M}f$.

Constructions of semiholonomic jets $\bar{J}^r$ and nonholonomic jets $\tilde{J}^r$ can be also interpreted as bundle functors on $\mathcal{FM}_m$ or on $\mathcal{M}f_m \times \mathcal{M}f$, see Section 1. The aim of this paper is to study natural transformations

$$\bar{J}^r \to J^r \quad \text{and} \quad \tilde{J}^r \to J^r$$

which will be called in short *symmetrizations* of semiholonomic or nonholonomic jets.

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We recall that the theory of higher order jets was established by C. Ehresmann [3], see also [4, 10, 12]. Now the theory of jets is a very powerful tool in many areas of differential geometry, mainly in the theory of connections, bundle functors and natural differential operators, see e.g. [1, 2, 5, 6, 9]. It is also well known that the theory of jets forms the theoretical background in calculus of variations, field theories and many areas of mathematical physics. We point out that the idea of symmetrization plays an important role not only in the theory of jets, but also in current physical theories. Moreover, the most important role in differential geometry and in mathematical physics is played by classical holonomic jets. That is why it is useful to study symmetrizations of higher order jets.

1. Preliminaries

In what follows, we denote by $\mathbb{N}$ the set of positive integers and by $\mathcal{FM}_{m,n}$ the category of fibered manifolds with m-dimensional bases, n-dimensional fibers and local fibered diffeomorphisms. All manifolds and maps are assumed to be infinitely differentiable.

We recall that the r-th *nonholonomic prolongation* $\widetilde{J}^r Y$ of a fibered manifold $Y \to M$ is defined by iteration

$$\widetilde{J}^1 Y = J^1 Y, \quad \widetilde{J}^r Y = J^1(\widetilde{J}^{r-1} Y \to M),$$

which yields natural identification $\widetilde{J}^r(\widetilde{J}^s Y) = \widetilde{J}^{r+s} Y$. Clearly, we have the canonical inclusion $J^r Y \subset \widetilde{J}^r Y$ given by $j^r_x s \mapsto j^1_x(u \mapsto j^{r-1}_u s)$ for every local section s of Y. The r-th *semiholonomic prolongation* $\bar{J}^r Y \to M$ is defined by the following induction. Write $\bar{J}^1 Y = J^1 Y$ and assume we have defined $\bar{J}^{r-1} Y \subset \widetilde{J}^{r-1} Y$ such that the restriction of $\beta_{\widetilde{J}^{r-2} Y} : \widetilde{J}^{r-1} Y \to \widetilde{J}^{r-2} Y$ maps $\bar{J}^{r-1} Y$ into $\bar{J}^{r-2} Y$, where $\beta_Y : J^1 Y \to Y$ means the projection. Then we define

$$\bar{J}^r Y = \{U \in J^1 \bar{J}^{r-1} Y; \beta_{\bar{J}^{r-1} Y}(U) = J^1 \beta_{\widetilde{J}^{r-2} Y}(U) \in \bar{J}^{r-1} Y\}.$$

Obviously, J^r, $\bar{J}^r$ and $\widetilde{J}^r$ are bundle functors on $\mathcal{FM}_m$ and for $r > 1$ we have $J^r Y \subset \bar{J}^r Y \subset \widetilde{J}^r Y$. Denoting by (x^i, y^p) the canonical coordinates on Y, the induced coordinates on $J^1 Y$ are $y^p_i = \frac{\partial y^p}{\partial x^i}$. The canonical coordinates on $\widetilde{J}^r Y$ can be introduced by the following induction. First, assume we have the coordinates $(x^i, y^p_{i_1 \dots i_{r-1}})$ on $\widetilde{J}^{r-1} Y$, where $i_1, \dots, i_{r-1} \in \{0, 1, \dots, m\}$. Then the induced coordinates on $\widetilde{J}^r Y$ are

$$x^i, \quad y^p_{i_1 \dots i_{r-1} 0} = y^p_{i_1 \dots i_{r-1}}, \quad y^p_{i_1 \dots i_{r-1} i} = \frac{\partial}{\partial x^i} y^p_{i_1 \dots i_{r-1}}.$$

Next, the semiholonomic prolongation $\bar{J}^r Y$ can be characterized by the following condition: $y^p_{i_1 \dots i_r} = y^p_{j_1 \dots j_r}$ provided the sequences obtained from $(i_1, \dots, i_r)$ and

$(j_1, \dots, j_r)$ by deleting all zeros with preserving the order of nonzero indices coincide. So the local coordinates on $\bar{J}^r Y$ are $(x^i, y^p_{i_1 \dots i_s})$, $s = 0, \dots, r$. Finally, the holonomic prolongation $J^r Y$ is characterized by full symmetry in all subscripts.

Every bundle functor F on $\mathcal{FM}_m$ induces a bundle functor F_0 on the product category $\mathcal{M}f_m \times \mathcal{M}f$ by $F_0(M, N) = F(M \times N \to M)$. Using such a point of view, the bundles of r-th semiholonomic or nonholonomic jets on $\mathcal{M}f_m \times \mathcal{M}f$ are defined as the r-th semiholonomic or nonholonomic prolongation of the product fibered manifold $M \times N \to M$, respectively,

$$\bar{J}^r_0(M, N) = \bar{J}^r(M \times N \to M), \quad \tilde{J}^r_0(M, N) = \tilde{J}^r(M \times N \to M).$$

In what follows, we omit the subscript 0 at the jet functors on $\mathcal{M}f_m \times \mathcal{M}f$, i.e. we write $\bar{J}^r(M, N) = \bar{J}^r_0(M, N)$, $\tilde{J}^r(M, N) = \tilde{J}^r_0(M, N)$.

Clearly, the jet functors J^r, $\bar{J}^r$ and $\tilde{J}^r$ on $\mathcal{FM}_m$ preserve fiber products and the corresponding induced jet functors on $\mathcal{M}f_m \times \mathcal{M}f$ preserve products in the second factor. By [7], every fiber product preserving bundle functor F on $\mathcal{FM}_m$ of order r can be characterized by a triple (A, H, t), where A is a Weil algebra of order r, $H : G^r_m \to \mathrm{Aut}A$ is a group homomorphism of the r-jet group in dimension m into the group of algebra automorphisms of A and $t : \mathbb{D}^r_m \to A$ is an equivariant algebra homomorphism, $\mathbb{D}^r_m = J^r_0(\mathbb{R}^m, \mathbb{R})$. According to [7], bundle functors G on $\mathcal{M}f_m \times \mathcal{M}f$ of order r in the first factor and preserving products in the second factor are characterized by pairs (A, H), where A and H are as above. Moreover, if $F = (A, H, t)$ is a functor on $\mathcal{FM}_m$, then the induced functor F_0 on $\mathcal{M}f_m \times \mathcal{M}f$ is of the form $F_0 = (A, H)$.

We recall that an r-th order nonholonomic connection on a fibered manifold $Y \to M$ is a section $\Gamma : Y \to \tilde{J}^r Y$. Such a connection is called semiholonomic or holonomic, if it has values in $\bar{J}^r Y$ or $J^r Y$, respectively. For $r = 1$ we obtain the concept of a general connection $Y \to J^1 Y$. Further, the product of two connections $\Gamma_1 : Y \to \tilde{J}^r Y$ and $\Gamma_2 : Y \to \tilde{J}^s Y$ is a connection $\Gamma_1 * \Gamma_2 : Y \to \tilde{J}^{r+s} Y$ defined by $\Gamma_1 * \Gamma_2 := \tilde{J}^s \Gamma_1 \circ \Gamma_2$. Finally, Ehresmann prolongation of a connection $\Gamma : Y \to J^1 Y$ is the r-th order connection $\Gamma^{(r-1)} : Y \to \tilde{J}^r Y$ defined by $\Gamma^{(1)} := \Gamma * \Gamma$, $\Gamma^{(r-1)} := \Gamma^{(r-2)} * \Gamma$. It is well known that Ehresmann prolongation $\Gamma^{(r-1)}$ is semiholonomic.

2. Symmetrization of jets on $\mathcal{FM}_{m,n}$

By a symmetrization of semiholonomic jets $\bar{J}^r Y$ we understand an $\mathcal{FM}_{m,n}$-natural transformation $\bar{J}^r \to J^r$ being the identity map on $J^r Y$ for any $Y \in \mathrm{Obj}\, \mathcal{FM}_{m,n}$. Quite analogously, we can define the symmetrization of nonholonomic jets $\tilde{J}^r Y$ and also the symmetrization of jets $\bar{J}^r(M, N)$ and $\tilde{J}^r(M, N)$.

First, for $r = 2$, we have a well-known symmetrization of second order semiholonomic jets

$$C^{(2)} : \bar{J}^2 Y \to J^2 Y, \quad (x^i, y^p, y^p_i, y^p_{ij}) \mapsto (x^i, y^p, y^p_i, y^p_{ij} + y^p_{ji}).$$

In the general case, we proved in [1]

Theorem 1. *All $\mathcal{FM}_{m,n}$-natural transformations $A : \bar{J}^r Y \to J^r Y$ are of the form*

(1) *For $m = 1$ and arbitrary natural r, $A = \mathrm{id} : \bar{J}^r Y = J^r Y \to J^r Y$.*
(2) *For $r = 1$ and $m \geq 2$, $A = \mathrm{id} : \bar{J}^1 Y = J^1 Y \to J^1 Y$.*
(3) *For $r = 2$ and $m \geq 2$, $A = C^{(2)} : \bar{J}^2 Y \to J^2 Y$.*
(4) *For $r \geq 3$ and $m \geq 2$ there is no $\mathcal{FM}_{m,n}$-natural transformation A in question.*

Proposition 1. *For $r \geq 2$ and $m \geq 2$ there is no $\mathcal{FM}_{m,n}$-natural transformation $\tilde{J}^r Y \to J^r Y$.*

By Theorem 1 and Proposition 1, to define symmetrizations of higher order semiholonomic and nonholonomic jets, it is unavoidable to use an additional geometric object. Using a projectable classical linear connection Σ on Y, we defined natural transformations

$$C^{(r)}(\Sigma) : \bar{J}^r Y \to J^r Y \quad \text{and} \quad s^{(r)}(\Sigma) : \tilde{J}^r Y \to J^r Y \quad \text{for any } r \tag{1}$$

depending on Σ, see [1]. We also proved that $C^{(2)}(\Sigma) = C^{(2)}$, which means that $C^{(r)}(\Sigma)$ generalizes $C^{(2)}$ for $r \geq 3$.

Moreover, the second author [11] introduced the symmetrization of semiholonomic jets $\bar{J}^r Y$ by means of a classical linear connection ∇ on the base manifold. He solved a more general problem on the existence of natural transformations of two fiber product preserving bundle functors on $\mathcal{FM}_m$. Given two fiber product preserving bundle functors $F = (A, H, t)$ and $F^1 = (A^1, H^1, t^1)$ of order r on $\mathcal{FM}_m$, he first defined a quasimorphism $\nu : (A, H, t) \to (A^1, H^1, t^1)$ as a $\mathrm{GL}(m)$-invariant homomorphism $\nu : A \to A^1$ such that $t^1 = \nu \circ t$. Then he proved that there is an $\mathcal{FM}_{m,n}$-natural transformation $FY \to F^1 Y$ depending on a classical linear connection on the base of Y if and only if there is a quasi-morphism $(A, H, t) \to (A^1, H^1, t^1)$. In particular, the Weil algebra of J^r is $\mathbb{D}^r_m = J^r_0(\mathbb{R}^m, \mathbb{R})$ and the Weil algebra of $\bar{J}^r$ is $\bar{\mathbb{D}}^r_m = \bar{J}^r_0(\mathbb{R}^m, \mathbb{R})$, see also (3). By [11], the usual symmetrization $\bar{\mathbb{D}}^r_m \to \mathbb{D}^r_m$ is a quasimorphism of the corresponding triples. This yields that given a classical linear connection ∇ on M, there is an $\mathcal{FM}_{m,n}$-natural transformation

$$S^{(r)}(\nabla) : \bar{J}^r Y \to J^r Y \quad \text{for any } r \tag{2}$$

depending on ∇. In [11], it is also proved that for $r \geq 2$ it is not possible to symmetrize nonholonomic jets $\tilde{J}^r Y$ by means of a classical linear connection on the base of Y.

3. Symmetrization of jets on $\mathcal{M}f_m \times \mathcal{M}f_n$

We start with several auxiliary assertions. From Proposition 3 in [1], we obtain directly

Proposition 2. *Let $m, n, r \in \mathbb{N}$, $m \geq 2$ and $r \geq 3$. Then there is no $\mathcal{M}f_m \times \mathcal{M}f_n$-natural operator A transforming connections $\Gamma : M \times N \to J^1(M \times N)$ on $\mathrm{pr}_1 : M \times N \to M$ into r-th order holonomic connections $A(\Gamma) : M \times N \to J^r(M \times N)$ extending Γ (i. e., such that $\pi^r_1 \circ A(\Gamma) = \Gamma$, where $\pi^r_1 : J^r(M \times N) \to J^1(M \times N)$ is the jet projection).*

Write

$$\bar{A}^r = \bar{J}^r_0(\mathbb{R}^m, \mathbb{R}) = \oplus^r_{k=0} \otimes^k \mathbb{R}^{m*}, \quad A^r = J^r_0(\mathbb{R}^m, \mathbb{R}) = \oplus^r_{k=0} S^k \mathbb{R}^{m*}. \tag{3}$$

Lemma 1. *Let $C_1, C_2 : \times^n \bar{A}^r \to A^q$ be $\mathrm{GL}(m)$-invariant maps such that $C_{1|\times^n A^r} = C_{2|\times^n A^r}$. Then $C_1 = C_2$*

Proof. For $n = 2$ and $q = r$ this is exactly Lemma 2 in [11]. The proof for any n and q is a direct modification of the proof of Lemma 2 from [11]. □

Proposition 3. *Let $m, n, r, q \in \mathbb{N}$. Let $D_1, D_2 : \bar{J}^r \to J^q$ be $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformations such that $D_{1|J^r(M,N)} = D_{2|J^r(M,N)}$. Then $D_1 = D_2$.*

Proof. Define $E_i : \times^n \bar{A}^r = \bar{J}^r_0(\mathbb{R}^m, \mathbb{R}^n) \to \times^n A^q = J^q_0(\mathbb{R}^m, \mathbb{R}^n)$ as the restrictions of $D_i : \bar{J}^r(\mathbb{R}^m, \mathbb{R}^n) \to J^q(\mathbb{R}^m, \mathbb{R}^n)$, $i = 1, 2$. Because of the $\mathcal{M}f_m \times \mathcal{M}f_n$-invariance of D_1 and D_2 it remains to show that $\mathrm{pr}_l \circ E_1 = \mathrm{pr}_l \circ E_2$ for $l = 1, \dots, n$, where $\mathrm{pr}_l : \times^n A^q \to A^q$ is the projection on the lth factor. But from the assumption of our proposition, we claim that $E_1, E_2 : \times^n \bar{A}^r \to \times^n A^q$ are $\mathrm{GL}(m)$-invariant maps such that $E_{1|\times^n A^r} = E_{2|\times^n A^r}$. So the proposition is an immediate consequence of Lemma 1. □

Proposition 4. *Let $m, n, r \in \mathbb{N}$, $m \geq 2$ and $r \geq 3$. There is no $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformation $A : \bar{J}^r \to J^r$ such that $A_{|J^r(M,N)} = \mathrm{id}$ for any $(M, N) \in \mathrm{Obj}(\mathcal{M}f_m \times \mathcal{M}f_n)$.*

Proof. Suppose that such A exists. We modify the respective part of the proof of Theorem 3 from [1]. Given a connection $\Gamma : M \times N \to J^1(M \times N)$ on $\mathrm{pr}_1 : M \times N \to M$, we have an r-th order holonomic connection $A(\Gamma) := A \circ \Gamma^{(r-1)} : M \times N \to J^r(M \times N) = J^r(M, N)$, where $\Gamma^{(r-1)} : M \times N \to \bar{J}^r(M \times N) = \bar{J}^r(M, N)$ is the r-th Ehresmann prolongation of Γ. Next, we have two $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformations $D_1 = \pi^r_1 \circ A : \bar{J}^r \to J^1$ and $D_2 = \bar{\pi}^r_1 : \bar{J}^r \to J^1$, where $\bar{\pi}^r_1$ and $\pi^r_1 : J^r \to J^1$ are jet projections. By the assumption of A, we see $D_{1|J^r(M,N)} = D_{2|J^r(M,N)}$. By Proposition 3 we have $D_1 = D_2$. Then $\pi^r_1 \circ A = \bar{\pi}^r_1$, so that $\pi^r_1 \circ A(\Gamma) = \pi^r_1 \circ A \circ \Gamma^{(r-1)} = \bar{\pi}^r_1 \circ \Gamma^{(r-1)} = \Gamma$. This yields that connection $A(\Gamma)$ extends Γ. But this is impossible because of Proposition 2. □

The book [6] proves

Proposition 5. *For $r \geq 2$ the only $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformations $A : J^r \to J^r$ are the identity and the contraction. For $r = 1$ we have*

$$J^1(M, N) = \mathrm{Hom}(TM, TN),$$

and all $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformations $J^1 \to J^1$ form the one-parameter family of homotheties $X \mapsto cX$, $c \in \mathbb{R}$.

The main result of this section is

Theorem 2. (1) *For $r = 1$ and $m, n \in \mathbb{N}$, all $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformations $\bar{J}^1 = J^1 \to J^1$ form the one-parameter family of homotheties.*
(2) *For $m = 1$, $r \geq 2$ and $n \in \mathbb{N}$, the only $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformations $\bar{J}^r = J^r \to J^r$ are the identity and the contraction.*
(3) *For $m \geq 2$, $r = 2$ and $n \in \mathbb{N}$, the only $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformation $\bar{J}^2 \to J^2$ are the classical symmetrization $\bar{J}^2 \to J^2$ of semiholonomic 2-jets and the trivial one $w \mapsto j^2_x([y])$, $w \in \bar{J}^2_x(M, N)_y$, $x \in M$, $y \in N$, where $[y] : M \to \{y\} \subset N$ denotes the constant map.*
(4) *For $m \geq 2$, $r \geq 3$ and $n \in \mathbb{N}$, the only $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformation $\bar{J}^r \to J^r$ is the trivial one $w \mapsto j^r_x([y])$, $w \in \bar{J}^r_x(M, N)_y$, $x \in M$, $y \in N$.*

Proof. The parts (1) and (2) are immediate consequences of Proposition 5. Part (3) follows from Propositions 3 and 5. Indeed, by these propositions we have that the number of such natural transformations is at least 2. Quite similarly, part (4) is the consequence of Propositions 3, 5 and 4. □

Proposition 6. *For $m \geq 2$, $r \geq 2$ and $n \in \mathbb{N}$, the only $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformation $\tilde{J}^r \to J^r$ is the contraction.*

Proof. For $r \geq 3$ the statement follows from Theorem 2 and the case $r = 2$ is examined in [8]. □

Using the naturality with respect to $\mathcal{M}f_m \times \mathcal{M}f$, we obtain easily from Theorem 2 the following

Corollary 1. (1) *For $r = 1$ and $m \in \mathbb{N}$, all $\mathcal{M}f_m \times \mathcal{M}f$-natural transformations $\bar{J}^1 = J^1 \to J^1$ form the one-parameter family of homotheties.*
(2) *For $m = 1$ and $r \geq 2$, the only $\mathcal{M}f_m \times \mathcal{M}f$-natural transformations $\bar{J}^r \to J^r$ are the identity and the contraction.*
(3) *For $m \geq 2$ and $r = 2$, the only $\mathcal{M}f_m \times \mathcal{M}f$-natural transformation $\bar{J}^2 \to J^2$ are the classical symmetrization $\bar{J}^2 \to J^2$ of semiholonomic 2-jets and the trivial one $w \mapsto j^2_x([y])$, $w \in \bar{J}^2_x(M, N)_y$, $x \in M$, $y \in N$, where $[y] : M \to \{y\} \subset N$ denotes the constant map.*
(4) *For $m \geq 2$ and $r \geq 3$, the only $\mathcal{M}f_m \times \mathcal{M}f$-natural transformation $\bar{J}^r \to J^r$ is the trivial one $w \mapsto j^r_x([y])$, $w \in \bar{J}^r_x(M, N)_y$, $x \in M$, $y \in N$.*

Corollary 2. *For $m \geq 2$ and $r \geq 2$, the only $\mathcal{M}f_m \times \mathcal{M}f$-natural transformation $\tilde{J}^r \to J^r$ is the contraction.*

In the rest of this section, we show that it is possible to symmetrize higher order semiholonomic jets on $\mathcal{M}f_m \times \mathcal{M}f_n$ by means of a classical linear connection ∇ on M. Indeed, we define easily the symmetrization of semiholonomic jets

$$S_0^{(r)}(\nabla) : \bar{J}^r(M, N) \to J^r(M, N) \quad \text{for any } r$$

depending on a classical linear connection ∇ on M. Let $S^{(r)}(\nabla) : \bar{J}^r(M \times N) \to J^r(M \times N)$ be the symmetrization (2) for the product fibered manifold $M \times N \to M$. This is exactly the $\mathcal{M}f_m \times \mathcal{M}f_n$-natural transformation $S_0^{(r)}(\nabla)$

$$\bar{J}^r(M, N) = \bar{J}^r(M \times N \to M) \overset{S^{(r)}(\nabla)}{\longrightarrow} J^r(M \times N \to M) = J^r(M, N).$$

By [11], for $r \geq 2$ it is not possible to define symmetrization $\tilde{J}^r Y \to J^r Y$ of nonholonomic jets on $\mathcal{F}\mathcal{M}_m$ by means of a classical linear connection ∇ on M. On the other hand, we have the $\mathcal{M}f_m \times \mathcal{M}f_n$-natural operator

$$s_0^{(r)} : \tilde{J}^r(M, N) \to J^r(M, N), \quad w \mapsto j_x^r([y]), \quad w \in \tilde{J}_x^r(M, N)_y, x \in M, y \in N.$$

REFERENCES

[1] M. Doupovec and W. M. Mikulski, "Holonomic extension of connections and symmetrization of jets," *Rep. Math. Phys.*, vol. 60(2), pp. 299–316, 2007.

[2] M. Doupovec and W. M. Mikulski, "On iteration of higher order jets and prolongation of connecions," *Ann. Polon. Math.*, vol. 100(1), pp. 55–75, 2011.

[3] C. Ehresmann, "Extension du calcul des jets aut jets non holonomes," *C.R.A.S. Paris*, vol. 239, pp. 1762–1764, 1954.

[4] I. Kolář, "A general point of view to nonholonomic jet bundles," *Cahiers Topol. Geom. Diff. Categoriques*, vol. 44, pp. 149–160, 2003.

[5] I. Kolář, "Weil Bundles as Generalized Jet Spaces," in *Handbook of Global Analysis*. Elsevier, 2008, pp. 625–664.

[6] I. Kolář, P. W. Michor, and J. Slovák, *Natural Operations in Differential Geometry*. Springer Verlag, 1993.

[7] I. Kolář and W. M. Mikulski, "On the fiber product preserving bundle functors," *Diff. Geom. Appl.*, vol. 11, pp. 105–111, 1999.

[8] I. Kolář and G. Vosmanská, "Natural operations with second order jets," *Supplemento ai Rendiconti del Circolo Matematico di Palermo, Serie II*, vol. 14, pp. 179–186, 1987.

[9] D. Krupka and J. Janyška, *Lectures on Differential Invariants*. Brno: University J. E. Purkyně, 1990.

[10] P. Libermann, "Introduction to the theory of semiholonomic jets," *Arch. Math. (Brno)*, vol. 33, pp. 173–189, 1977.

[11] W. M. Mikulski, "On symmetrization of jets," *Czechoslovak Math. J.*, vol. 61(1), pp. 157–168, 2011.

[12] D. Saunders, *The Geometry of Jet Bundles*, ser. London Mathematical Society Lecture Note Series. Cambridge University Press, 1989, vol. 142.

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