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ON FUZZY *P*-CONTINUOUS MULTIFUNCTIONS

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ABSTRACT. In this paper, by using the fuzzy property P, some characterisations and properties of certain types of upper (lower) fuzzy continuous multifunctions including upper (lower) fuzzy continuous, upper (lower) fuzzy almost continuous, upper (lower) fuzzy c-continuous, upper (lower) fuzzy almost c-continuous, upper (lower) fuzzy c^*-continuous, upper (lower) fuzzy s-continuous, upper (lower) fuzzy almost s-continuous, upper (lower) fuzzy ℓ-continuous, upper (lower) fuzzy almost ℓ-continuous functions are given.

Mathematics Subject Classification: 54A40, 03E72

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1. INTRODUCTION

It is well known that several types of fuzzy upper (lower) continuous multifunctions are given in literature. By using property P, our main goal here is to give some characterisations and properties of certain types of upper (lower) fuzzy continuous multifunctions including upper (lower) fuzzy continuous, upper (lower) fuzzy almost continuous, upper (lower) fuzzy c-continuous, upper (lower) fuzzy almost c-continuous, upper (lower) fuzzy c^*-continuous, upper (lower) fuzzy s-continuous, upper (lower) fuzzy almost s-continuous, upper (lower) fuzzy ℓ-continuous, upper (lower) fuzzy almost ℓ-continuous functions.

Fuzzy sets on a universe X will be denoted by Greek letters as μ, ρ, η, etc. Fuzzy points will be denoted by x_ε, y_ν, etc. For any fuzzy points x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ iff $\varepsilon \leq \mu(x)$. A fuzzy set x_ε is called quasi-coincident with a fuzzy set ρ, denoted by $x_\varepsilon \mathrel{q} \rho$, iff $\varepsilon + \rho(x) > 1$. A fuzzy set μ is called quasi-coincident with a fuzzy set ρ, denoted by $\mu \mathrel{q} \rho$, iff there exists a $x \in X$ such that $\mu(x)+\rho(x) > 1$. In this paper we use the concept of a fuzzy topological space as introduced by [1]. By $\mathrm{int}(\mu)$, $\mathrm{cl}(\mu)$ and $\mathrm{co}(\mu)$, we mean the interior of μ, the closure of μ, and the complement of μ.

Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space X to a fuzzy topological space Y. For any fuzzy set $\mu \leq X$, $F^+(\mu)$ and $F^-(\mu)$ are defined by $F^+(\mu) = \{x \in X : F(x) \leq \mu\}$, $F^-(\mu) = \{x \in X : F(x) \mathrel{q} \mu\}$. We know that $F^-(\mathrm{co}(\beta)) = \mathrm{co}(F^+(\beta))$ for any fuzzy set $\beta \leq Y$ [8].

2. Fuzzy P-continuous multifunctions

Definition 1. Let (X, τ) be a fuzzy topological space and let $\mu \leq X$ be a fuzzy set. Then it is said that

(i) μ is a fuzzy P-set if μ possesses fuzzy property P,
(ii) μ has fuzzy P-complement if $\mathrm{co}(\mu)$ possesses fuzzy property P.

The following definition of fuzzy P-continuity is considered for fuzzy multisetting from [6] in the classical sense.

Definition 2. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). Then it is said that F is fuzzy lower (upper) P-continuous if for each $x_\varepsilon \in X$ and for each fuzzy set μ having a P-complement such that $x_\varepsilon \in F^-(\mu)$ ($x_\varepsilon \in F^+(\mu)$), there exists an open fuzzy set ρ containing x_ε such that $\rho \leq F^-(\mu)$ ($\rho \leq F^+(\mu)$).

The following table gives us the list of some types of lower (upper) fuzzy P-continuous multifunctions with property P.

The definitions of lower (upper) fuzzy c-continuous, lower (upper) fuzzy almost c-continuous, lower (upper) fuzzy c^*-continuous, lower (upper) fuzzy s-continuous, lower (upper) fuzzy ℓ-continuous and lower (upper) fuzzy almost ℓ-continuous multifunctions are considered for fuzzy multisetting from [2, 3, 10, 4, 5] and [7], respectively.

The definitions of lower (upper) fuzzy continuous and lower (upper) fuzzy almost continuous multifunctions are considered from [8] and [9], respectively.

n°	Fuzzy P-set	Fuzzy lower P-con.	Fuzzy upper P-con.
1.	f. closed	f. l. con.	f. u. con.
2.	f. regular closed	f. l. almost con.	f. u. almost con.
3.	f. closed compact	f. l. c-con.	f. u. c-con.
4.	f. regular closed compact	f. l. almost c-con.	f. u. almost c-con.
5.	f. closed countable compact	f. l. c^*-con.	f. u. c^*-con.
6.	f. closed connected	f. l. s-con.	f. u. s-con.
7.	f. regular closed connected	f. l. almost s-con.	f. u. almost s-con.
8.	f. closed Lindelöf	f. l. ℓ -con.	f. u. ℓ -con.
9.	f. regular closed Lindelöf	f. l. almost ℓ-con.	f. u. almost ℓ-con.

We know that a net $(x^\alpha_{\varepsilon_\alpha})$ in a fuzzy topological space (X, τ) is said to be eventually in the fuzzy set $\mu \leq X$ if there exists an index $\alpha_0 \in J$ such that $x^\alpha_{\varepsilon_\alpha} \in \mu$ for all $\alpha \geq \alpha_0$.

The following theorem gives us some characterisation of fuzzy lower (upper) P-continuous multifunctions.

Theorem 1. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). Then the following statements are equivalent:*

(i) *F is a lower (upper) fuzzy P-continuous multifunction.*

(ii) *For each $x_\varepsilon \in X$ and for each fuzzy set μ having a P-complement such that $F(x_\varepsilon)\ \mathrm{q}\ \mu$ ($F(x_\varepsilon) \leq \mu$), there exists an open fuzzy set ρ containing x_ε such that if $y_\beta \in \rho$, then $F(y_\beta)\ \mathrm{q}\ \mu$ ($F(y_\beta) \leq \mu$).*

(iii) *$F^-(\mu)$ ($F^+(\mu)$) is an open fuzzy set for any fuzzy set $\mu \leq Y$ having a P-complement.*

(iv) *$F^+(\mathrm{co}(\mu))$ ($F^-(\mathrm{co}(\mu))$) is a closed fuzzy set for any fuzzy set $\mu \leq Y$ having a P-complement.*

(v) *For each $x_\varepsilon \in X$ and for each net $(x^\alpha_{\varepsilon_\alpha})$ which converges to x_ε in X and for each fuzzy set $\rho \leq Y$ having a P-complement such that $x_\varepsilon \in F^-(\rho)$ ($x_\varepsilon \in F^+(\rho)$), the net $(x^\alpha_{\varepsilon_\alpha})$ is eventually in $F^-(\rho)$ ($F^+(\rho)$).*

Proof. (i)$\Leftrightarrow$(ii). This statement is obvious.

(i)$\Leftrightarrow$(iii). Let $x_\varepsilon \in F^-(\mu)$ and let μ be a fuzzy set having a P-complement. From (i), there exists an open fuzzy set ρ containing x_ε such that $\rho \leq F^-(\mu)$. It follows that $x_\varepsilon \in \mathrm{int}(F^-(\mu))$ and (iii) holds.

The converse can be shown easily.

(iii)$\Leftrightarrow$(iv). Since $F^-(\mathrm{co}(\mu)) = \mathrm{co}(F^+(\mu))$ and $F^+(\mathrm{co}(\mu)) = \mathrm{co}(F^-(\mu))$, the proof is clear.

(i)$\Rightarrow$(v). Let $x^\alpha_{\varepsilon_\alpha}$ be a net which converges to x_ε in X and let $\rho \leq Y$ be any fuzzy set having a P-complement such that $x_\varepsilon \in F^-(\rho)$. Since F is a lower fuzzy P-continuous multifunction, it follows that there exists an open fuzzy set $\mu \leq X$ containing x_ε such that $\mu \leq F^-(\rho)$. Since $x^\alpha_{\varepsilon_\alpha}$ converges to x_ε, it follows that there exists an index $\alpha_0 \in J$ such that $x^\alpha_{\varepsilon_\alpha} \in \mu$ for all $\alpha \geq \alpha_0$. From here, we obtain that $x^\alpha_{\varepsilon_\alpha} \in \mu \leq F^-(\rho)$ for all $\alpha \geq \alpha_0$. Thus, the net $(x^\alpha_{\varepsilon_\alpha})$ is eventually in $F^-(\rho)$.

(v)$\Rightarrow$(i). Suppose that (i) is not true. There exists a point x_ε and a fuzzy set μ having a P-complement with $x_\varepsilon \in F^-(\mu)$ such that $\rho \nleq F^-(\mu)$ for each open fuzzy set $\rho \leq X$ containing x_ε. Let $x_{\varepsilon_\rho} \in \rho$ and $x_{\varepsilon_\rho} \notin F^-(\mu)$ for each open fuzzy set $\rho \leq X$ containing x_ε. Then for the neighbourhood net (x_{ε_ρ}), $x_{\varepsilon_\rho} \to x_\varepsilon$, but (x_{ε_ρ}) is not eventually in $F^-(\mu)$. This is a contradiction. Thus, F is a fuzzy lower (upper) P-continuous multifunction.

The proof of the fuzzy upper P-continuity of F is similar to that given above. $\square$

Example. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). Then the following statements are equivalent to Theorem 1 when we take fuzzy P-set as fuzzy closed compact set:

(i) F is a lower (upper) fuzzy c-continuous multifunction,

(ii) For each $x_\varepsilon \in X$ and for each fuzzy open set μ having a compact complement such that $F(x_\varepsilon)$ q μ ($F(x_\varepsilon) \leq \mu$), there exists an open fuzzy set ρ containing x_ε such that if $y_\beta \in \rho$, then $F(y_\beta)$ q μ ($F(y_\beta) \leq \mu$),

(iii) $F^-(\mu)$ ($F^+(\mu)$) is an open fuzzy set for any fuzzy open set $\mu \leq Y$ having a compact complement,

(iv) $F^+(1-\mu)$ ($F^-(1-\mu)$) is a closed fuzzy set for any fuzzy open set $\mu \leq Y$ having a compact complement,

(v) For each $x_\varepsilon \in X$ and for each net $(x^\alpha_{\varepsilon_\alpha})$ which converges to x_ε in X and for each fuzzy open set $\rho \leq Y$ having a compact complement such that $x_\varepsilon \in F^-(\rho)$ ($x_\varepsilon \in F^+(\rho)$), the net $(x^\alpha_{\varepsilon_\alpha})$ is eventually in $F^-(\rho)(F^+(\rho))$.

Definition 3. Suppose that (X, τ), (Y, υ) and (Z, ω) are fuzzy topological spaces. If $F_1 : X \to Y$ and $F_2 : Y \to Z$ are fuzzy multifunctions, then the fuzzy multifunction $F_2 \circ F_1 : X \to Z$ is defined by $(F_2 \circ F_1)(x_\varepsilon) = F_2(F_1(x_\varepsilon))$.

Theorem 2. *Let (X, τ), (Y, υ), (Z, ω) be fuzzy topological spaces and let $F : X \to Y$ and $G : Y \to Z$ be fuzzy multifunctions. If $F : X \to Y$ is an upper fuzzy continuous multifunction and $G : Y \to Z$ is an upper fuzzy P-continuous multifunction, then $G \circ F : X \to Z$ is an upper fuzzy P-continuous multifunction.*

Proof. Let $\mu \leq Z$ be any fuzzy set having a P-complement. From the definition of $G \circ F$, we have $(G \circ F)^+(\mu) = F^+(G^+(\mu))$. Since G is an upper fuzzy P-continuous multifunction, it follows that $G^+(\mu)$ is an open fuzzy set. Since F is an upper fuzzy continuous multifunction, it follows that $F^+(G^+(\mu))$ is an open fuzzy set. This shows that $G \circ F$ is an upper fuzzy P-continuous multifunction. □

Theorem 3. *Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). If F is a lower (upper) fuzzy P-continuous multifunction and $\mu \leq X$ is a fuzzy set, then the restriction multifunction $F|_\mu : \mu \to Y$ is a lower (upper) fuzzy P-continuous multifunction.*

Proof. Suppose that $\beta \leq Y$ is a fuzzy set having a P-complement. Let $x_\varepsilon \in \mu$ and let $x_\varepsilon \in F^-|_\mu(\beta)$. Since F is a lower fuzzy P-continuous multifunction, it follows that there exists a fuzzy open set $x_\varepsilon \in \rho$ such that $\rho \leq F^-(\beta)$. From here we obtain that $x_\varepsilon \in \rho \wedge \mu$ and $\rho \wedge \mu \leq F^-|_\mu(\beta)$. Thus, we show that the restriction multifunction $F|_\mu$is lower fuzzy P-continuous.

The proof of the fuzzy upper P-continuity of $F|_\mu$ is similar to the above. □

Theorem 4. *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). Let $\{\gamma_\alpha : \alpha \in \Phi\}$ be an open cover of X. If the restriction multifunction $F_\alpha = F_{\gamma_\alpha}$ is a lower (upper) fuzzy P-continuous multifunction for each $\alpha \in \Phi$, then F is lower (upper) fuzzy P-continuous.*

Proof. Let $\mu \leq Y$ be a fuzzy set having a P-complement. Since F_α is lower fuzzy P-continuous for each α, from Theorem 1, $F^-_\alpha(\mu) \leq \mathrm{int}_{\gamma_\alpha}(F^-_\alpha(\mu))$ and from here

$F^{-}(\mu)\wedge\gamma_\alpha \leq \mathrm{int}_{\gamma_\alpha}(F^{-1}(\mu)\wedge\gamma_\alpha)$ and $F^{-}(\mu)\wedge\gamma_\alpha \leq \mathrm{int}(F^{-1}(\mu))\wedge\gamma_\alpha$. Since $\{\gamma_\alpha : \alpha \in \Phi\}$ is an open cover of X, it follows that $F^{-}(\mu) \leq \mathrm{int}(F^{-}(\mu))$. Thus from Theorem 1, we obtain that F is a lower fuzzy P-continuous multifunction.

The proof of the fuzzy upper P-continuity of F is similar to the above. □

Definition 4. Suppose that $F : X \to Y$ is a fuzzy multifunction from a fuzzy topological space X to a fuzzy topological space Y. The fuzzy graph multifunction $G_F : X \to X \times Y$ of F is defined as $G_F(x_\varepsilon) = \{x_\varepsilon\} \times F(x_\varepsilon)$ [8].

Theorem 5. *Suppose that a finite product of fuzzy P-sets is a fuzzy P-set and X is a fuzzy P-set. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). If the graph function of F is lower (upper) fuzzy P-continuous, then F is a lower (upper) fuzzy P-continuous multifunction.*

Proof. For the fuzzy sets $\beta \leq X$, $\eta \leq Y$, we take

$$(\beta \times \eta)(z, y) = \begin{cases} 0 & \text{if } z \notin \beta \\ \eta(y) & \text{if } z \in \beta. \end{cases}$$

Let $x_\varepsilon \in X$ and let $\mu \leq Y$ be a fuzzy set having a P-complement such that $x_\varepsilon \in F^{-}(\mu)$. We obtain that $x_\varepsilon \in G_F^{-}(X \times \mu)$. Since fuzzy graph multifunction G_F is fuzzy lower P-continuous and X is a fuzzy P-set, it follows that there exists an open fuzzy set $\rho \leq X$ containing x_ε such that $\rho \leq G_F^{-}(X \times \mu)$. From here, we obtain that $\rho \leq F^{-}(\mu)$. Thus, F is a fuzzy lower P-continuous multifunction.

The proof of the fuzzy upper P-continuity of F is similar to the above. □

Theorem 6. *Suppose that (X, τ) and (X_α, τ_α) are fuzzy topological spaces and X_α is a fuzzy P-set where $\alpha \in J$. Let $F : X \to \prod_{\alpha\in J} X_\alpha$ be a multifunction from X to the product space $\prod_{\alpha\in J} X_\alpha$ and let $P_\alpha : \prod_{\alpha\in J} X_\alpha \to X_\alpha$ be the projection multifunction for each $\alpha \in J$ which is defined by $P_\alpha((x_\alpha)) = \{x_\alpha\}$ and let a product of fuzzy P-sets be a fuzzy P-set. If F is a fuzzy upper P-continuous multifunction, then $P_\alpha \circ F$ is a fuzzy upper P-continuous multifunction for each $\alpha \in J$.*

Proof. Take any $\alpha_0 \in J$. Let $x_\varepsilon \in X$ and let $\mu_{\alpha_0} \leq X_{\alpha_0}$ be a fuzzy set having a P-complement such that $x_\varepsilon \in (P_{\alpha_0} \circ F)^{+}(\mu_{\alpha_0})$. We know that $x_\varepsilon \in (P_{\alpha_0} \circ F)^{+}(\mu_{\alpha_0}) = F^{+}(P^{+}_{\alpha_0}(\mu_{\alpha_0})) = F^{+}(\mu_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$. Since F is a fuzzy upper P-continuous multifunction and a product of fuzzy P-sets is a fuzzy P-set, it follows that there exists an open fuzzy set $\rho \leq X$ containing x_ε such that $\rho \leq F^{+}\left(\mu_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha\right) = F^{+}(P^{+}_{\alpha_0}(\mu_{\alpha_0})) = (P_{\alpha_0} \circ F)^{+}(\mu_{\alpha_0})$. This shows that $P_{\alpha_0} \circ F$ is a fuzzy upper P-continuous multifunction.

Thus we obtain that $P_\alpha \circ F$ is a fuzzy upper P-continuous multifunction for each $\alpha \in J$. □

Theorem 7. *Suppose that for each $\alpha \in J$, (X_α, τ_α), $(Y_\alpha, \upsilon_\alpha)$ are fuzzy topological spaces and Y_α possesses property P for each $\alpha \in J$. Let $F_\alpha : X_\alpha \to Y_\alpha$ be a* *multifunction for each $\alpha \in J$ and let $F : \prod_{\alpha\in J} X_\alpha \to \prod_{\alpha\in J} Y_\alpha$ be defined by $F((x_\alpha)) = \prod_{\alpha\in J} F_\alpha(x_\alpha)$ from the product space $\prod_{\alpha\in J} X_\alpha$ to the product space $\prod_{\alpha\in J} Y_\alpha$. If F is an upper fuzzy P-continuous multifunction and the product of fuzzy P-sets is a fuzzy P-set, then each F_α is an upper fuzzy P-continuous multifunction for each $\alpha \in J$.*

Proof. Let $\mu_\alpha \leq Y_\alpha$ be a fuzzy set having a P-complement. Then $\mu_\alpha \times \prod_{\alpha\neq\beta} Y_\beta$ is a fuzzy set having a P-complement. Since F is an upper fuzzy P-continuous multifunction, it follows that $F^+(\mu_\alpha \times \prod_{\alpha\neq\beta} Y_\beta) = F^+(\mu_\alpha) \times \prod_{\alpha\neq\beta} X_\beta$ is an open fuzzy set. Consequently, we obtain that $F^+(\mu_\alpha)$ is an open fuzzy set. Thus, we show that F_α is an upper fuzzy P-continuous multifunction. □

Theorem 8. *Let $F : X \to Y$ be a fuzzy function from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, υ). Suppose that Y has a base of neighbourhood such that the complement of each fuzzy set of the base of neighbourhood is the finite union of P-sets. If F is lower fuzzy P-continuous multifunction, then F is lower fuzzy continuous.*

Proof. Let $x_\varepsilon \in X$ and let μ be any fuzzy open set such that $x_\varepsilon \in F^-(\mu)$. Then there exists a fuzzy point $y_\nu \in \mu$ such that $x_\varepsilon \in F^-(y_\nu)$. Since Y has a base of neighbourhood such that the complement of each fuzzy set of the base of neighbourhood is finite unions of P-sets, it follows that there exists a neighbourhood β of y_ν such that $\beta \leq \mu$ and $\mathrm{co}(\beta) = \bigvee_{i=1}^n \eta_i$ where each η_i is a P-set. From here we obtain that $\beta = \bigwedge_{i=1}^n \mathrm{co}(\eta_i)$. Then $y_\nu \in \mathrm{co}(\eta_i)$ and $x_\varepsilon \in F^-(\mathrm{co}(\eta_i))$ for each $i = 1, 2, ..., n$. Since F is a lower fuzzy P-continuous multifunction, it follows that there exists an open set ρ_i containing x_ε such that $\rho_i \leq F^-(\mathrm{co}(\eta_i))$. We take $\rho = \bigwedge_{i=1}^n \rho_i$, then $\rho = \bigwedge_{i=1}^n \rho_i \leq \bigwedge_{i=1}^n F^-(\mathrm{co}(\eta_i)) = F^-(\bigwedge_{i=1}^n \mathrm{co}(\eta_i)) = F^-(\beta) \leq F^-(\mu)$. Thus, we obtain that F is a lower fuzzy P-continuous multifunction. □

Theorem 9. *Suppose that (X_1, τ_1), (X_2, τ_2), (Y_1, υ_1) and (Y_2, υ_2) are fuzzy topological spaces and $F_1 : X_1 \to Y_1$, $F_2 : X_2 \to Y_2$ are fuzzy multifunctions and suppose that $\eta \times \beta$ is a fuzzy P-set iff η and β are fuzzy P-sets for any fuzzy sets $\eta \leq Y_1$, $\beta \leq Y_2$. Let $F_1 \times F_2 : X_1 \times X_2 \to Y_1 \times Y_2$ be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\varepsilon, y_\nu) = F_1(x_\varepsilon) \times F_2(y_\nu)$. Then $F_1 \times F_2$ is an upper fuzzy P-continuous multifunction iff F_1 and F_2 are upper fuzzy P-continuous multifunctions.*

Proof. We know that $(\mu^* \times \beta^*)(x_\varepsilon, y_\nu) = \min\{(\mu^*(x), \beta^*(y)\}$ for any fuzzy sets μ^*, β^* and for any fuzzy points x_ε, y_ν.

Let $\mu \times \beta \leq Y_1 \times Y_2$ be a fuzzy set having a P-complement. We know that $(F_1 \times F_2)^+(\mu \times \beta) = F_1^+(\mu) \times F_2^+(\beta)$ and $\rho \times \xi$ is a fuzzy P-set iff ρ and ξ are fuzzy P-sets for any fuzzy sets $\rho \leq Y_1$, $\xi \leq Y_2$. From Theorem 1, the proof is obtained. □

Theorem 10. *Suppose that (X, τ), (Y, υ), (Z, ω) are fuzzy topological spaces and $F_1 : X \to Y$, $F_2 : X \to Z$ are fuzzy multifunctions and suppose that $\eta \times \beta$ is a fuzzy P-set iff η and β are fuzy P-sets for any fuzzy sets $\eta \leq Y$, $\beta \leq Z$. Let $F_1 \times F_2 : X \to Y \times Z$ be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\varepsilon) = F_1(x_\varepsilon) \times F_2(x_\varepsilon)$. Then*

$F_1 \times F_2$ is an upper fuzzy P-continuous multifunction iff F_1 and F_2 are upper fuzzy P-continuous multifunctions.

Proof. ($\Rightarrow$) Let $x_\varepsilon \in X$ and let $\mu \leq Y, \beta \leq Z$ be fuzzy sets having P-complements such that $x_\varepsilon \in F_1^+(\mu)$ and $x_\varepsilon \in F_2^+(\beta)$. Then we obtain that $F_1(x_\varepsilon) \leq \mu$ and $F_2(x_\varepsilon) \leq \beta$ and from here, $F_1(x_\varepsilon) \times F_2(x_\varepsilon) = (F_1 \times F_2)(x_\varepsilon) \leq \mu \times \beta$. We have $x_\varepsilon \in (F_1 \times F_2)^+(\mu \times \beta)$. Since $F_1 \times F_2$ is an upper fuzzy P-continuous multifunction, it follows that there exists an open fuzzy set ρ containing x_ε such that $\rho \leq (F_1 \times F_2)^+(\mu \times \beta)$. We obtain that $\rho \leq F_1^+(\mu)$ and $\rho \leq F_2^+(\beta)$. Thus we obtain that F_1 and F_2 are upper fuzzy P-continuous multifunctions.

($\Leftarrow$) Let $x_\varepsilon \in X$ and let $\mu \times \rho \leq Y \times Z$ be a fuzzy set having a P-complement such that $x_\varepsilon \in (F_1 \times F_2)^+(\mu \times \rho)$. We obtain that $(F_1 \times F_2)(x_\varepsilon) = F_1(x_\varepsilon) \times F_2(x_\varepsilon) \leq \mu \times \rho$. From here we have $F_1(x_\varepsilon) \leq \mu$ and $F_2(x_\varepsilon) \leq \rho$ and then $x_\varepsilon \in F_1^+(\mu)$ and $x_\varepsilon \in F_2^+(\rho)$. Since F_1 and F_2 are upper fuzzy P-continuous multifunctions, it follows that there exist fuzzy open sets η containing x_ε and β containing x_ε such that $\eta \leq F_1^+(\mu)$ and $\beta \leq F_2^+(\rho)$. We take $x_\varepsilon \in \eta \wedge \beta$. Then we obtain that $\eta \wedge \beta \leq F_1^+(\mu)$ and $\eta \wedge \beta \leq F_2^+(\rho)$. Hence we have $\eta \wedge \beta \leq (F_1 \times F_2)^+(\mu \times \rho)$. Thus, we show that $F_1 \times F_2$ is an upper fuzzy P-continuous multifunction. □

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