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SOME RESULTS ON ONE-SIDED GENERALIZED LIE IDEALS WITH DERIVATION

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Abstract. Let R be a prime ring with a characteristic not equal to two, σ, τ be automorphisms of R, and d be a nonzero derivation of R commuting with σ and τ. It is proved that for any (σ, τ)-left Lie ideal U of R: (1) if $d(U) \subseteq Z$, then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$, (2) if $d^2(U) = 0$, then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$, (3) if $char\, R \neq 2, 3$, $d(U) \subseteq U$ and $d^2(U) \subseteq Z$, then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$.

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1. Introduction

Let R be a ring and σ, τ be two mappings from R into itself. We write $[x, y]$, $[x, y]_{\sigma,\tau}$ for $xy - yx$ and $x\sigma(y) - \tau(y)x$, respectively, and make extensive use of basic commutator identities: $(xy, z) = x[y, z] + (x, z)y = x(y, z) - [x, z]y$, $[xy, z]_{\sigma,\tau} = x[y, z]_{\sigma,\tau} + [x, \tau(z)]y = x[y, \sigma(z)] + [x, z]_{\sigma,\tau}y$.

An additive mapping $D : R \to R$ is called a *derivation* if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$. A derivation D is *inner* if there exists an $a \in R$ such that $D(x) = [a, x]$ holds for all $x \in R$..

For subsets $A, B \subset R$, let $[A, B]$ ($[A, B]_{\sigma,\tau}$) be the additive subgroup generated by all $[a, b]$ ($[a, b]_{\sigma,\tau}$) for all $a \in A$ and $b \in B$. We recall that in a *Lie ideal*, L is an additive subgroup of R such that $[R, L] \subset L$. We first introduce the generalized Lie ideal in [6] as follows. Let U be an additive subgroup of R, $\sigma, \tau : R \to R$ two mappings. Then (i) U is a $(\sigma, \tau)-$right Lie ideal of R if $[U, R]_{\sigma,\tau} \subset U$. (ii) U is a $(\sigma, \tau)-$left Lie ideal of R if $[R, U]_{\sigma,\tau} \subset U$. (iii) U is both a $(\sigma, \tau)-$right Lie ideal and $(\sigma, \tau)-$left Lie ideal of R then U is a $(\sigma, \tau)-$Lie ideal of R. Every Lie ideal of R is a $(1, 1)-$left Lie ideal of R, where $1 : R \to R$ is the identity map. As an example, let I be the set of integers,

$$R = \left\{ \begin{pmatrix} x & y \\ z & t \end{pmatrix} \mid x, y, z, t \in I \right\},$$

$$U = \left\{ \begin{pmatrix} x & y \\ 0 & x \end{pmatrix} \mid x, y \in I \right\} \subset R,$$

and $\sigma, \tau : R \to R$ the mappings defined by $\tau(x) = axa$, $\sigma(x) = bxb^{-1}$, where $a = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \in R$. Then U is a $(\sigma, \tau)-$left Lie ideal but not a Lie ideal of R. Some algebraic properties of $(\sigma, \tau)-$Lie ideals are considered in [2], [3] and [6], where further references can be found.

Let R be a prime ring with a characteristic not equal to two, $d : R \to R$ a nonzero derivation of R and U a Lie ideal of R. In [5] Bergen at all state that if $d^2(U) = 0$, then $U \subset Z$. Lee and Lee extended this result that if $d^2(U) \subset Z$, then $U \subset Z$ in [4]. Let d be a nonzero derivation such that $\sigma d = d\sigma, \tau d = d\tau$ and U a $(\sigma, \tau)-$Lie ideal of R. Aydın and Soytürk [3] proved that if $d^2(U) = 0$, then $U \subset Z$. In the present paper, we generalize this result on $(\sigma, \tau)-$left Lie ideal of R. Furthermore, we shall extend this theorem by proving that $d^2(U) \subset Z$ then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$ in the case of a characteristic not equal to two and three.

Throughout, R will represent a prime ring with a characteristic not equal to 2 with automorphisms σ, τ and non-zero derivation d such that $\sigma d = d\sigma, \tau d = d\tau$ and Z the center of R, U a $(\sigma, \tau)-$left Lie ideal of R. Further, we often use the relations:

$$[xy, z]_{\sigma,\tau} = x[y, z]_{\sigma,\tau} + [x, \tau(z)]y = x[y, \sigma(z)] + [x, z]_{\sigma,\tau}y.$$

2. **Results**

Lemma 1. *Let U a $(\sigma, \tau)-$left Lie ideal of R. $d^2(U) = 0$ and $d(U) \subset Z$ then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$.*

Proof. If $U \subset Z$, then the proof is obvious. So, we assume that $U \not\subset Z$. For any $u \in U$ and $x \in R$, $\tau(u)[x, u]_{\sigma,\tau} = [\tau(u)x, u]_{\sigma,\tau} + [\tau(u), \tau(u)]x \in U$. By hypothesis, $0 = d^2(\tau(u)[x, u]_{\sigma,\tau}) = d(d(\tau(u))[x, u]_{\sigma,\tau} + \tau(u)d([x, u]_{\sigma,\tau})) = 2d(\tau(u))d([x, u]_{\sigma,\tau})$. Since $char R \neq 2$, we obtain $d(\tau(u))d([x, u]_{\sigma,\tau}) = 0$, for all $x \in R, u \in U$. Because of $d(U) \subset Z$ we have,

$$d(u) = 0 \qquad or \qquad d([x, u]_{\sigma,\tau}) = 0 \qquad \forall x \in R, u \in U. \tag{2.1}$$

Assume $d(u) \neq 0$. Then $d([x, u]_{\sigma,\tau}) = 0$, for all $x \in R$. Writing $x\sigma(u)$ by x in this equation, $0 = d([x\sigma(u), u]_{\sigma,\tau}) = d([x, u]_{\sigma,\tau}\sigma(u)) = d([x, u]_{\sigma,\tau})\sigma(u) + [x, u]_{\sigma,\tau}d(\sigma(u))$

we obtain

$$[x,u]_{\sigma,\tau}d(\sigma(u))=0 \qquad \forall x\in R. \tag{2.2}$$

Substituting $xy, y\in R$ for x in (2.2), we have $0=[xy,u]_{\sigma,\tau}d(\sigma(u))=x[y,u]_{\sigma,\tau}d(\sigma(u))+[x,\tau(u)]yd(\sigma(u))$ and so,

$$[R,\tau(u)]Rd(\sigma(u))=0.$$

By primeness of R, we obtain $u\in Z$. Thus, if we return to (2.1), then we get

$$d(u)=0 \qquad \textit{or} \qquad u\in Z.$$

Now, let us define the subsets $L=\{u\in U \mid u\in Z\}$ and $K=\{u\in U \mid d(u)=0\}$. Clearly, each L and K is an additive subgroup of U. Moreover, U is the set-theoretic union of L and K. But a group cannot be the set-theoretic union of two proper subgroups, hence $L=U$ or $K=U$. In the former case, $U\subset Z$, which is a contradiction. Therefore, it must be $d(U)=0$ and so,

$$0=d([x,u]_{\sigma,\tau})=[d(x),u]_{\sigma,\tau} \qquad \text{for all } x\in R,\ u\in U.$$

By [7, Lemma 1], we obtain $\sigma(u)+\tau(u)\in Z$, for all $u\in U$. Hence the proof is complete. □

Theorem 1. *Let U a $(\sigma,\tau)-$left Lie ideal of R. If $d(U)\subset Z$ then $\sigma(u)+\tau(u)\in Z$, for all $u\in U$.*

Proof. Assume that $U\not\subset Z$. For any $x,y\in R$ and $u,v\in U$, by hypothesis, $d([d(v)x,u]_{\sigma,\tau})$ $=d(d(v)[x,u]_{\sigma,\tau}+[d(v),\tau(u)]x)=d(d(v)[x,u]_{\sigma,\tau})\in Z$ and so,

$$d^2(v)[x,u]_{\sigma,\tau}+d(v)d([x,u]_{\sigma,\tau})\in Z$$

Since Z is a subring of R and $d(U)\subset Z$, we have

$$d^2(v)[x,u]_{\sigma,\tau}\in Z \qquad \forall x\in R, u,v\in U. \tag{2.3}$$

Replacing x by $x\sigma(u), u\in U$ in (2.3) and applying the above argument, we obtain

$$d^2(v)[x,u]_{\sigma,\tau}\sigma(u)\in Z \qquad \forall x\in R, u,v\in U.$$

Since $d^2(v)[x,u]_{\sigma,\tau}\in Z$ and R is prime ring, we get

$$d^2(v)[x,u]_{\sigma,\tau}=0 \qquad \textit{or} \qquad u\in Z.$$

If $d^2(v)[x,u]_{\sigma,\tau}=0$ for all $x\in R$. In this equation by taking $xy, y\in R$ for x and using this equation, we have $0=d^2(v)[xy,u]_{\sigma,\tau}=d^2(v)[x,u]_{\sigma,\tau}y+d^2(v)x[y,\sigma(u)]=d^2(v)x[y,\sigma(u)]$. By the primeness of R, it implies that $d^2(U)=0$ or $U\subset Z$. In the former case, we get $\sigma(u)+\tau(u)\in Z$, for all $u\in U$ by Lemma 1. Thus, we conclude that $\sigma(u)+\tau(u)\in Z$, for all $u\in U$. □

Now, suppose that U is a $(\sigma,\tau)-$left Lie ideal of R. Since for all $u,v\in U$ and $x\in R$,

$$\begin{aligned}[x,d(u)+v]_{\sigma,\tau}&=[x,d(u)]_{\sigma,\tau}+[x,v]_{\sigma,\tau}\\&=[x,d(u)]_{\sigma,\tau}+[d(x),u]_{\sigma,\tau}-[d(x),u]_{\sigma,\tau}+[x,v]_{\sigma,\tau}\\&=d([x,u]_{\sigma,\tau})-[d(x),u]_{\sigma,\tau}+[x,v]_{\sigma,\tau}\in d(U)+U.\end{aligned}$$

We conclude that $d(U)+U$ is a $(\sigma,\tau)-$left Lie ideal of R. Furthermore, if $d^2(U)=0$ then $d(d(U)+U)\subset d(U)\subset d(U)+U$ and $d^2(d(U)+U)=0$. Therefore without losing generality, we may assume that if U is a $(\sigma,\tau)-$left Lie ideal of such that $d^2(U)=0$, then $d(U)\subset U$.

Lemma 2. *Let U a $(\sigma,\tau)-$left Lie ideal of R. $d^2(U)=0$ and a be an element of R. If $ad([R,U]_{\sigma,\tau})=0$, then $a=0$ or $\sigma(u)+\tau(u)\in Z$, for all $u\in U$.*

Proof. For $x[\sigma(u),\sigma(u)]+[x,u]_{\sigma,\tau}\sigma(u)=[x\sigma(u),u]_{\sigma,\tau}\in[R,U]_{\sigma,\tau}$ by hypothesis $0=ad([x,u]_{\sigma,\tau}\sigma(u))=ad([x,u]_{\sigma,\tau})\sigma(u)+a[x,u]_{\sigma,\tau}d(\sigma(u))$ and so

$$a[x,u]_{\sigma,\tau}d(\sigma(u))=0,\forall x\in R,u\in U. \tag{2.4}$$

Since $d^2(U)=0$, from the above remark we may assume $d(U)\subset U$. So, replacing $u+d(v),v\in U$ by u in (2.4)

$$0=a[x,u+d(v)]_{\sigma,\tau}d(\sigma(u+d(v)).$$

Expanding the last equation and using $d^2(U)=0,\sigma d=d\sigma$ and (2.4), we get $a[x,d(v)]_{\sigma,\tau}d(\sigma(u))=0$, for all $u,v\in U,x\in R$. That is,

$$\sigma^{-1}(a[x,d(v)]_{\sigma,\tau})d(U)=0\,.$$

By [1, Theorem 2] we have $\sigma(u)+\tau(u)\in Z$, for all $u\in U$ or $a[x,d(v)]_{\sigma,\tau}=0$. Replacing $xy,y\in R$ in the last equation, we obtain $ax[y,\sigma(d(v)]=0$. Since R is a prime ring, we conclude $a=0$ or $d(U)\subset Z$. It gives $\sigma(u)+\tau(u)\in Z$, for all $u\in U$ from Theorem 1. This completes the proof. □

Theorem 2. *Let U a $(\sigma,\tau)-$left Lie ideal of R. If $d^2(U)=0$ then $\sigma(u)+\tau(u)\in Z$, for all $u\in U$.*

Proof. Assume that $U\nsubseteq Z$. There exists a $u_0\in U$ such that

$$\sigma(u_0)+\tau(u_0)\notin Z. \tag{2.5}$$

For $[x,u]_{\sigma,\tau}\sigma(u)\in U$,

$$\begin{aligned}0&=d^2([x,u]_{\sigma,\tau}\sigma(u))\\&=d^2([x,u]_{\sigma,\tau})\sigma(u)+2d([x,u]_{\sigma,\tau})d(\sigma(u))+[x,u]_{\sigma,\tau}d^2(\sigma(u)).\end{aligned}$$

In view of the hypothesis and $charR\neq 2$, we have

$$d([x,u]_{\sigma,\tau})d(\sigma(u))=0,\forall x\in R,u\in U. \tag{2.6}$$

Similarly for $\tau(u)[x,u]_{\sigma,\tau}\in U$, we get

$$d(\tau(u))d([x,u]_{\sigma,\tau})=0,\forall x\in R,u\in U. \tag{2.7}$$

By hypothesis $0=d^2([u,v]_{\sigma,\tau})=[d^2(u),v]_{\sigma,\tau}+2[d(u),d(v)]_{\sigma,\tau}+[u,d^2(v)]_{\sigma,\tau}$. Using $d^2(U)=0$ and $charR\neq 2$, we obtain

$$[d(u),d(v)]_{\sigma,\tau}=0,\forall u,v\in U.$$

That is

$$d(u)\sigma(d(v))=\tau(d(v))d(u),\forall u,v\in U. \tag{2.8}$$

Now, let us linearize (2.7) on $u = u + v$ and use (2.8), then we have

$$d(\tau(u))d([x,v]_{\sigma,\tau} + d(\tau(v))d([x,u]_{\sigma,\tau}) = 0, \forall x \in R, u, v \in U. \tag{2.9}$$

Multiply on the right by $d(\sigma(u))$ and use (2.8), (2.6), we obtain

$$(d(\tau(u)))^2 d([x,v]_{\sigma,\tau}) = 0, \forall x \in R, u, v \in U.$$

The last equation reduces to $(d(\tau(U)))^2 d([R,U]_{\sigma,\tau}) = 0$. By Lemma 2 and (2.5), we get $(d(U))^2 = 0$. Otherwise, writing $d(v)$ for v in (2.9) and using $d\tau = \tau d$, we see that

$$d(U)\tau^{-1}([d(x), d(v)]_{\sigma,\tau}) = 0, \forall x \in R, v \in U.$$

This means from [1, Theorem 2] $\sigma(u)+\tau(u) \in Z$, for all $u \in U$ or $[d(x), d(v)]_{\sigma,\tau} = 0$. By our assumption, we get $[d(x), d(v)]_{\sigma,\tau} = 0$, for all $x \in R, v \in U$. If we write $xd(u), u \in U$ for x in the last equation, we have $0 = [d(xd(u), d(v)]_{\sigma,\tau} = [d(x)d(u), d(v)]_{\sigma,\tau} = [d(x), \tau(d(v)]d(u)$ and so,

$$[d(R), \tau(d(U))]d(U) = 0.$$

From the above argument, we have $d(U) \subset Z$ by [1, Theorem 2]. That is $\sigma(u)+\tau(u) \in Z$, for all $u \in U$ from Theorem 1. □

Theorem 3. *Let U a $(\sigma,\tau)-$left Lie ideal of R and $charR \neq 2,3$. If $d(U) \subset U$ and $d^2(U) \subset Z$, then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$.*

Proof. If $U \subset Z$, then the proof of the theorem is obvious. So, we assume that $U \not\subseteq Z$. That is,

$$\sigma(u_0) + \tau(u_0) \notin Z, \ \exists u_0 \in U. \tag{2.10}$$

Suppose that $d(Z) = 0$. Thus, we have

$$d^3(U) = d(d^2(U)) \subset d(Z) = 0.$$

Now, for $\tau(u)[x,u]_{\sigma,\tau} \in U$, where $x \in R$ and $u \in U$,

$$\begin{aligned} 0 &= d^3(\tau(u)[x,u]_{\sigma,\tau}) \\ &= 3(d^2(\tau(u))d([x,u]_{\sigma,\tau}) + d(\tau(u))d^2([x,u]_{\sigma,\tau}). \end{aligned}$$

Since $charR \neq 3$, we get

$$d^2(\tau(u))d([x,u]_{\sigma,\tau}) + d(\tau(u))d^2([x,u]_{\sigma,\tau}) = 0.$$

Taking $d(u)$ by u and using $\tau d = d\tau, d^3(U) = 0$, we obtain

$$d^2(\tau(u))d^2([x,d(u)]_{\sigma,\tau}) = 0.$$

Since $d^2(U) \subset Z$, the last equation gives us

$$d^2(u) = 0 \qquad or \qquad d^2([x,d(u)]_{\sigma,\tau}) = 0.$$

Let us define $K = \{u \in U \mid d^2(u) = 0\}$ and $L = \{u \in U \mid d^2([x,d(u)]_{\sigma,\tau}) = 0, \forall x \in R\}$. Clearly, both K and L are additive subgroups of U. Moreover, U is the set-theoretic union of K and L. But a group cannot be the set-theoretic union of two proper subgroups, hence $K = U$ or $L = U$. If $K = U$ then $\sigma(u) + \tau(u) \in Z$, for all $u \in U$ by Theorem 2 and it contradicts (2.10). So, we get $L = U$. That is,

$$d^2([x,d(u)]_{\sigma,\tau}) = 0, \forall x \in R, u \in U. \tag{2.11}$$

In this equation replace x by $\tau(d(u))x, u \in U, x \in R$, then we get

$$\begin{aligned} 0 &= d^2(\tau(d(u))[x,d(u)]_{\sigma,\tau}) \\ &= \tau(d^3(u))[x,d(u)]_{\sigma,\tau} + 2\tau(d^2(u))d([x,d(u)]_{\sigma,\tau}) + \tau(d(u))d^2([x,d(u)]_{\sigma,\tau}). \end{aligned}$$

Using (2.11) and $d^3(U) = 0$, $charR \neq 2$, we obtain $\tau(d^2(u))d([x,d(u)]_{\sigma,\tau}) = 0$. Since $d^2(U) \subset Z$, we have

$$d^2(u) = 0 \qquad or \qquad d([x,d(u)]_{\sigma,\tau}) = 0.$$

Let $K = \{u \in U \mid d^2(u) = 0\}$ and $L = \{u \in U \mid d([x,d(u)]_{\sigma,\tau}) = 0, \forall x \in R\}$. Each of K and L is an additive subgroup of U such that $U = K \cup L$. The above trick gives us $U = K$ or $U = L$. In the former case, $d^2(U) = 0$, which forces $\sigma(u) + \tau(u) \in Z$, for all $u \in U$ by Theorem 2, which is a contradiction. Thus $U = L$ and hence $d([x,d(u)]_{\sigma,\tau}) = 0$ for all $u \in U$. Replacing $\tau(d(u))x, u \in U, x \in R$ by x we have $\tau(d^2(u))[x,d(u)]_{\sigma,\tau} = 0$. Since $d^2(U) \subset Z$, we obtain

$$d^2(u) = 0 \qquad or \qquad [x,d(u)]_{\sigma,\tau} = 0 \text{ for all } x \in R. \tag{2.12}$$

Again applying the above trick, we obtain $[x,d(u)]_{\sigma,\tau} = 0$. Taking $xy, y \in R$ in place of x and using (2.12), we have

$$0 = [xy,d(u)]_{\sigma,\tau} = x[y,d(u)]_{\sigma,\tau} + [x,\sigma(d(u))]y = [x,\sigma(d(u))]y\,.$$

Since R is a prime ring, we obtain $d(U) \subset Z$. By Theorem 1, it gives $\sigma(u)+\tau(u) \in Z$, for all $u \in U$, which is a contradiction. Thus, in the case of $d(Z) = 0$ the proof is completed.

Now, we would like to settle the problem when $d(Z)$ is different from zero. There is a non-zero $d(\alpha) \in d(Z)$ such that $\alpha \in Z$. In view of the hypothesis for $[\alpha x,u]_{\sigma,\tau} = \alpha[x,u]_{\sigma,\tau} \in U$,

$$d^2(\alpha[x,u]_{\sigma,\tau}) = d^2(\alpha)[x,u]_{\sigma,\tau} + 2d(\alpha)d([x,u]_{\sigma,\tau}) + \alpha d^2([x,u]_{\sigma,\tau}) \in Z\,.$$

Since $d^2(U) \subset Z$, the third term is in the center of R. So, we get

$$d^2(\alpha)[x,u]_{\sigma,\tau} + 2d(\alpha)d([x,u]_{\sigma,\tau}) \in Z, \forall x \in R, u \in U. \tag{2.13}$$

Replace x by $x\alpha$ in (2.13) to get

$$(d^2(\alpha)[x,u]_{\sigma,\tau} + 2d(\alpha)d([x,u]_{\sigma,\tau}))\alpha + 2d(\alpha)[x,u]_{\sigma,\tau}d(\alpha) \in Z.$$

However, in view of (2.13) and $\alpha \in Z$, this equation reduces to $2d(\alpha)[x,u]_{\sigma,\tau}d(\alpha) \in Z$. Since R is a prime ring, $charR \neq 2$ and $0 \neq d(\alpha) \in Z$, we have $[x,u]_{\sigma,\tau} \in Z$ for all $x \in R, u \in U$. By [8, Lemma 1], we obtain $\sigma(u) + \tau(u) \in Z$, for all $u \in U$. This completes the proof. □

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