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A GENERALIZATION OF ESSENTIAL SUPPLEMENTED LATTICES

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Abstract. In this work, all lattices are complete modular lattices with the smallest element 0 and the greatest element 1. Let L be a lattice. If every essential element of L has a weak supplement in L, then L is called a weakly essential supplemented (briefly, weakly e-supplemented) lattice. In this work, some properties of these lattices are investigated. The concept of weakly essential supplemented lattice is a generalization of the concept of essential supplemented lattice. Let L be a weakly e-supplemented lattice. Then $1/r(L)$ have no essential elements with distinct from 1. Let L be a lattice, $a_1, a_2, \ldots, a_n \in L$ and $1 = a_1 \vee a_2 \vee \cdots \vee a_n$. If $a_i/0$ is weakly e-supplemented for every $i = 1, 2, \ldots, n$, then L is also weakly e-supplemented. Let L be a weakly e-supplemented lattice and $a \in L$. Then the quotient sublattice $1/a$ is weakly e-supplemented. Let L be a lattice. Then L is weakly e-supplemented if and only if every essential element of L is β_* equivalent to a weak supplement in L.

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1. INTRODUCTION

Throughout this paper, all lattices are complete modular lattices with the smallest element 0 and the greatest element 1. Let L be a lattice, $a, b \in L$ and $a \leq b$. A sublattice $\{x \in L | a \leq x \leq b\}$ is called a *quotient sublattice*, denoted by b/a. An element a' of a lattice L is called a *complement* of a in L if $a \wedge a' = 0$ and $a \vee a' = 1$. In this case we say a and a' are *direct summands* of L and denoted by $1 = a \oplus a'$. A lattice L is said to be *complemented* if each element of L has at least one complement in L. An element c of L is said to be *compact* if for every subset X of L such that $c \leq \vee X$ there is a finite $F \subset X$ such that $c \leq \vee F$. A lattice L is said to be *compact* if 1 is compact. An element a of L is said to be *small* or *superfluous* in L and denoted by $a \ll L$ if $a \vee b \neq 1$ holds for every $b \neq 1$, or equivalently, $b = 1$ for every $b \in L$ with $a \vee b = 1$. An element a of L is said to be *essential* if $a \wedge b \neq 0$ holds for every $b \neq 0$ and denoted by $a \trianglelefteq L$. This equivalent to $a \wedge b = 0$ implies that $b = 0$. The meet of all maximal elements ($\neq 1$) of a lattice L is called the *radical* of L and denoted by $r(L)$. An element c of L is called a *supplement* of b in L if it is minimal for $b \vee c = 1$. a is a supplement of b in a lattice L if and only if $a \vee b = 1$ and $a \wedge b \ll a/0$. L is called a *supplemented* lattice if every element of L has a supplement in L. If every element of L has a supplement that is a direct summand of L, then L is called a $\oplus-$*supplemented* lattice. L is called an *essential supplemented* (briefly, *e-supplemented*) lattice if every essential element of L has a supplement in L. We say that an element b of L *lies above* an element a of L if $a \leq b$ and $b \ll 1/a$. L is said to be *hollow* if every element ($\neq 1$) is superfluous in L, and L is said to be *local* if L has the greatest element ($\neq 1$). An element a of L is called a *weak supplement* of b in L if $a \vee b = 1$ and $a \wedge b \ll L$. L is called a *weakly supplemented* lattice if every element of L has a weak supplement in L. An element $a \in L$ has *ample supplements* in L if for every $b \in L$ with $a \vee b = 1$, a has a supplement b' in L with $b' \leq b$. L is called an *amply supplemented* lattice if every element of L has ample supplements in L. L is called an *amply essential supplemented* (briefly, *amply e-supplemented*) lattice if every essential element of L has ample supplements in L. It is clear that every supplemented lattice is weakly supplemented and every amply supplemented lattice is supplemented. Let L be a lattice. It is defined β_* relation on the elements of L by $a\beta_* b$ with $a, b \in L$ if and only if for each $t \in L$ such that $a \vee t = 1$ then $b \vee t = 1$ and for each $k \in L$ such that $b \vee k = 1$ then $a \vee k = 1$.

More information about (amply) supplemented lattices are in [5, 11]. More details about weakly supplemented lattices are in [1]. More information about $\oplus-$ supplemented lattices are in [2]. More details about (amply) essential supplemented lattices are in [14, 19]. More information about (amply) supplemented modules are in [4, 7–10]. More information about weakly supplemented modules are in [6]. More details about (amply) essential supplemented modules are in [16, 17]. More details about weakly essential supplemented modules are in [12]. The definition of β_* relation on lattices and some properties of this relation are in [13]. This relation is a generalization of β^* relation on modules. The definition of β^* relation on modules and some properties of this relation are in [3, 18]. More details about lying above on lattices are in [5, 11, 13]. More details about lying above on modules are in [9, 10].

Lemma 1. *Let L be a lattice. The following assertions hold.*

(1) *If $a, b \in L$ and $a \leq b$, then $a \trianglelefteq L$ if and only if $a \trianglelefteq b/0$ and $b \trianglelefteq L$.*
(2) *Let $a, b \in L$ and $a \leq b$. If $b \trianglelefteq 1/a$, then $b \trianglelefteq L$.*
(3) *Let $a, b, c, d \in L$, $a \leq c$ and $b \leq d$. If $a \trianglelefteq c/0$ and $b \trianglelefteq d/0$, then $a \wedge b \trianglelefteq (c \wedge d)/0$.*
(4) *If $a \trianglelefteq L$ and $b \trianglelefteq L$, then $a \wedge b \trianglelefteq L$.*
(5) *If $a \trianglelefteq L$, then $a \wedge b \trianglelefteq b/0$ for every $b \in L$.*
(6) *If $a \trianglelefteq L$, then $a \vee b \trianglelefteq L$ for every $b \in L$.*

Proof. See [5]. □

2. Weakly essential supplemented lattices

Definition 1. Let L be a lattice. If every essential element of L has a weak supplement in L, then L is called a weakly essential supplemented (briefly, weakly e-supplemented) lattice. (See also [15]).

Clearly we can see that every essential supplemented lattice is weakly essential supplemented, but the converse of this statement is not true in general. (See Example 2). The concept of weakly essential supplemented lattice is a generalization of the concept of essential supplemented lattice.

Definition 2. Let L be a lattice and $x \in L$. If x is a weak supplement of an essential element of L, then x is called a weak e-supplement element in L.

Proposition 1. *Let L be a weakly e-supplemented lattice. If every element of L with distinct from 0 is essential in L, then L is weakly supplemented.*

Proof. Clear from definitions. □

Lemma 2. *Let L be a weakly e-supplemented lattice. Then $1/r(L)$ have no essential elements with distinct from 1.*

Proof. Let k be any essential element of $1/r(L)$. Since $k \trianglelefteq 1/r(L)$, by Lemma 1, $k \trianglelefteq L$ and since L is weakly e-supplemented, k has a weak supplement t in L. Then $1 = k \vee t$ and $k \wedge t \ll L$. Since $1 = k \vee t$, $1 = k \vee (t \vee r(L))$. Since $k \wedge t \ll L$, by [5, Lemma 7.6], $k \wedge t \leq r(L)$. Then $k \wedge (t \vee r(L)) = (k \wedge t) \vee r(L) = r(L)$ and $1 = k \oplus (t \vee r(L))$ in $1/r(L)$. Since $1 = k \oplus (t \vee r(L))$ in $1/r(L)$ and $k \trianglelefteq 1/r(L)$, $k = 1$. Hence $1/r(L)$ have no essential elements with distinct from 1. □

Corollary 1. *Let L be a weakly e-supplemented lattice. If $r(L) = 0$, then L have no essential elements with distinct from 1.*

Proof. Since $r(L) = 0$, $L = 1/0 = 1/r(L)$. Then by Lemma 2, L have no essential elements with distinct from 1. □

Corollary 2. *Let L be a weakly e-supplemented lattice, $k \trianglelefteq L$ and $k \vee r(L) \neq 1$. Then $k \vee r(L)$ is not essential in $1/r(L)$.*

Proof. Clear from Lemma 2. □

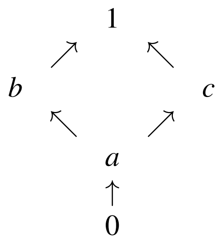
Corollary 3. *Let L be a weakly e-supplemented lattice, $k \trianglelefteq L$ and $r(L) \leq k$. Then k is not essential in $1/r(L)$.*

Proof. Clear from Corollary 2. □

Corollary 4. *Let L be an essential supplemented lattice, $k \trianglelefteq L$ and $r(L) \leq k$. Then k is not essential in $1/r(L)$.*

Proof. Clear from Corollary 3, since every essential supplemented lattice is weakly essential supplemented. □

Example 1. Consider the lattice $L=\{0,a,b,c,1\}$ given by the following diagram:



Here $r(L)=a$, $b\trianglelefteq L$ but $b\ntrianglelefteq 1/r(L)$. Here also $c\trianglelefteq L$ but $c\ntrianglelefteq 1/r(L)$.

Lemma 3. *Let L be a lattice and $r(L)\ll L$. If $a\vee r(L)$ is a direct summand of $1/r(L)$ for every essential element a of L, then L is weakly e-supplemented.*

Proof. Let $a\trianglelefteq L$. By hypothesis, $a\vee r(L)$ is a direct summand of $1/r(L)$ and there exists $x\in 1/r(L)$ such that $a\vee r(L)\vee x=1$ and $(a\vee r(L))\wedge x=r(L)$. Here $1=a\vee r(L)\vee x=a\vee x$ and $a\wedge x\leq (a\vee r(L))\wedge x=r(L)$. Since $r(L)\ll L$, $a\wedge x\ll L$. Hence x is a weak supplement of a in L and L is weakly e-supplemented. □

Corollary 5. *Let L be a compact lattice. If $a\vee r(L)$ is a direct summand of $1/r(L)$ for every essential element a of L, then L is weakly e-supplemented.*

Proof. Since L is compact, by [5, Lemma 7.8 (iii)], $r(L)\ll L$. Then by Lemma 3, L is weakly e-supplemented, as desired. □

Corollary 6. *Let L be a lattice and $r(L)\ll L$. If a is a direct summand of $1/r(L)$ for every essential element a of L with $r(L)\leq a$, then L is weakly e-supplemented.*

Proof. Let $x\trianglelefteq L$. By Lemma 1, $x\vee r(L)\trianglelefteq L$ and by hypothesis, $x\vee r(L)$ is a direct summand of $1/r(L)$. Then by Lemma 3, L is weakly e-supplemented, as desired. □

Corollary 7. *Let L be a compact lattice. If a is a direct summand of $1/r(L)$ for every essential element a of L with $r(L)\leq a$, then L is weakly e-supplemented.*

Proof. Since L is compact, by [5, Lemma 7.8 (iii)], $r(L)\ll L$. Then by Corollary 6, L is weakly e-supplemented, as desired. □

Lemma 4. *Let L be a lattice and $r(L)\ll L$. If $a\vee r(L)$ is a direct summand of $1/r(L)$ for every $a\in L$, then L is weakly supplemented.*

Proof. Similar to proof of Lemma 3. □

Corollary 8. *Let L be a compact lattice. If $a\vee r(L)$ is a direct summand of $1/r(L)$ for every $a\in L$, then L is weakly supplemented.*

Proof. Since L is compact, by [5, Lemma 7.8 (iii)], $r(L)\ll L$. Then by Lemma 4, L is weakly supplemented, as desired. □

Corollary 9. *Let L be a lattice and $r(L) \ll L$. If every element of $1/r(L)$ is a direct summand of $1/r(L)$, then L is weakly supplemented.*

Proof. Let $x \in L$. By hypothesis, $x \vee r(L)$ is a direct summand of $1/r(L)$. Then by Lemma 4, L is weakly supplemented, as desired. □

Corollary 10. *Let L be a compact lattice. If every element of $1/r(L)$ is a direct summand of $1/r(L)$, then L is weakly supplemented.*

Proof. Since L is compact, by [5, Lemma 7.8 (iii)], $r(L) \ll L$. Then by Corollary 9, L is weakly supplemented, as desired. □

Lemma 5. *Let L be a lattice, x be an essential element of L and $m \in L$. If $m/0$ is weakly e-supplemented and $x \vee m$ has a weak supplement in L, then x has a weak supplement in L.*

Proof. Let y be a weak supplement of $x \vee m$ in L. Then $1 = x \vee m \vee y$ and $(x \vee m) \wedge y \ll L$. Since $x \trianglelefteq L$, by Lemma 1, $(x \vee y) \trianglelefteq L$ and $(x \vee y) \wedge m \trianglelefteq m/0$. Since $m/0$ is weakly e-supplemented, $(x \vee y) \wedge m$ has a weak supplement z in $m/0$. This case $m = ((x \vee y) \wedge m) \vee z$ and $(x \vee y) \wedge z = (x \vee y) \wedge m \wedge z \ll m/0$. Then $1 = x \vee m \vee y = x \vee ((x \vee y) \wedge m) \vee z \vee y = x \vee y \vee z$ and $x \wedge (y \vee z) \leq ((x \vee y) \wedge z) \vee ((x \vee z) \wedge y) \leq ((x \vee y) \wedge z) \vee ((x \vee m) \wedge y) \ll L$. Hence $y \vee z$ is a weak supplement of x in L. □

Corollary 11. *Let L be a lattice, $x \trianglelefteq L$ and $m_1, m_2, \ldots, m_n \in L$. If $x \vee m_1 \vee m_2 \vee \cdots \vee m_n$ has a weak supplement in L and $m_i/0$ is weakly e-supplemented for every $i = 1, 2, \ldots, n$, then x has a weak supplement in L.*

Proof. Clear from Lemma 5. □

Lemma 6. *Let L be a lattice, $a_1, a_2 \in L$ and $1 = a_1 \vee a_2$. If $a_1/0$ and $a_2/0$ are weakly e-supplemented, then L is also weakly e-supplemented.*

Proof. Let $x \trianglelefteq L$. Then 0 is a weak supplement of $x \vee a_1 \vee a_2$ in L. Since $a_2/0$ is weakly e-supplemented and $x \vee a_1 \trianglelefteq L$, by Lemma 5, $x \vee a_1$ has a weak supplement in L. Since $a_1/0$ is weakly e-supplemented and $x \trianglelefteq L$, by Lemma 5, x has a weak supplement in L. Hence L is weakly e-supplemented. □

Corollary 12. *Let L be a lattice, $a_1, a_2, \ldots, a_n \in L$ and $1 = a_1 \vee a_2 \vee \cdots \vee a_n$. If $a_i/0$ is weakly e-supplemented for every $i = 1, 2, \ldots, n$, then L is also weakly e-supplemented.*

Proof. Clear from Lemma 6. □

Lemma 7. *Let L be a weakly e-supplemented lattice and $a \in L$. Then the quotient sublattice $1/a$ is weakly e-supplemented.*

Proof. Let $x \trianglelefteq 1/a$. Then by Lemma 1, $x \trianglelefteq L$ and since L is weakly e-supplemented, x has a weak supplement y in L. Since $a \leq x$, we can easily see that $a \vee y$ is a weak supplement of x in $1/a$. Hence $1/a$ is weakly e-supplemented. □

Corollary 13. *Let L be a weakly e-supplemented lattice. Then $a/0$ is weakly e-supplemented for every direct summand a of L.*

Proof. Let a be a direct summand of L. Then there exists $b \in L$ such that $a \oplus b = 1$. By Lemma 7, $1/b$ is weakly e-supplemented. Then by $1/b = (a \vee b)/b \cong a/(a \wedge b) = a/0$, $a/0$ is weakly e-supplemented. □

Lemma 8. *Let L be a lattice and $a, b, c \in L$. If $a \vee b = 1$ and $(a \wedge b) \vee c = 1$, then $a \vee (b \wedge c) = 1$ and $b \vee (a \wedge c) = 1$.*

Proof. See [13, Lemma 2]. □

Lemma 9. *Let L be a lattice. Then L is weakly e-supplemented if and only if every essential element of L is β_* equivalent to a weak supplement in L.*

Proof. ($\Longrightarrow$) Let L be a weakly e-supplemented lattice and a be any essential element of L. Since L is weakly e-supplemented, a has a weak supplement b in L. Then a is also a weak supplement of b in L. Since $a\beta_* a$ in L, a is β_* equivalent to a weak supplement in L.

($\Longleftarrow$) Let every essential element of L is β_* equivalent to a weak supplement in L. Let a be any essential element of L. By hypothesis, there exists a weak supplement element b in L with $a\beta_* b$. Let b be a weak supplement of c in L. Then c is a weak supplement of b in L and since $a\beta_* b$, by [13, Theorem 4], c is a weak supplement of a in L. Hence L is weakly e-supplemented. □

Corollary 14. *Let L be a lattice. Then L is weakly e-supplemented if and only if every essential element of L lies above a weak supplement in L.*

Proof. ($\Longrightarrow$) Let L be a weakly e-supplemented lattice and a be any essential element of L. Since L is weakly e-supplemented, a has a weak supplement b in L. Then a is also a weak supplement of b in L. Since a lies above a in L, a lies above a weak supplement in L.

($\Longleftarrow$) Clear from Lemma 9. But we prove this part as follows:

Let every essential element of L lies above a weak supplement in L. Let a be any essential element of L. By hypothesis, a lies above a weak supplement b in L. Let b be a weak supplement of c in L. Since b is a weak supplement of c in L, $b \vee c = 1$ and $b \wedge c \ll L$. Since a lies above b in L, $b \leq a$ and $a \ll 1/b$. Since $b \leq a$ and $b \vee c = 1$, $a \vee c = 1$. Let $(a \wedge c) \vee t = 1$ for $t \in L$. By Lemma 8, $a \vee (c \wedge t) = 1$. Then $a \vee (c \wedge t) \vee b = 1$ and since $a \ll 1/b$, $(c \wedge t) \vee b = 1$. By also Lemma 8, $(b \wedge c) \vee t = 1$. Since $(b \wedge c) \vee t = 1$ and $b \wedge c \ll L$, $t = 1$. Hence c is a weak supplement of a in L. Thus L is weakly e-supplemented. □

Corollary 15. *Let L be a lattice. Then L is weakly e-supplemented if and only if every essential element of L is a weak supplement in L.*

Proof. Clear from Lemma 9. □

Corollary 16. *Let L be a lattice. If every essential element of L is* β_* *equivalent to a weak e-supplement element in L, then L is weakly e-supplemented.*

Proof. Clear from Lemma 9. □

Corollary 17. *Let L be a lattice. If every essential element of L lies above a weak e-supplement element in L, then L is weakly e-supplemented.*

Proof. Clear from Corollary 16. □

Lemma 10. *Let L be a lattice. If every element of L has a weak supplement that is a supplement element in L, then L is supplemented.*

Proof. Let $a \in L$. By hypothesis, a has weak supplement b that is a supplement element in L. Here $a \vee b = 1$ and $a \wedge b \ll L$. Since $a \wedge b \ll L$ and b is a supplement element in L, by [11, Lemma 10], $a \wedge b \ll b/0$ and b is a supplement of a in L. Hence L is supplemented, as desired. □

Corollary 18. *Let L be a lattice. If every element of L has a weak supplement that is a direct summand of L, then L is* $\oplus-$*supplemented. (See also [2, Proposition 2]).*

Proof. Clear from Lemma 10. □

Lemma 11. *Let L be a lattice. If every essential element of L has a weak supplement that is a supplement element in L, then L is essential supplemented.*

Proof. Similar to proof of Lemma 10. □

Corollary 19. *Let L be a lattice. If every essential element of L has a weak supplement that is a direct summand of L, then L is essential supplemented.*

Proof. Clear from Lemma 11. □

Example 2. Let Γ be a family all submodules of the $\mathbb{Z}-$module ${}_{\mathbb{Z}}\mathbb{Q}$. It is clear that Γ is a lattice by the operation $\subset$. Here for $A, B \in \Gamma$, $A \vee B = A + B$ and $A \wedge B = A \cap B$. By [4, Example 20.12], ${}_{\mathbb{Z}}\mathbb{Q}$ is weakly supplemented but not supplemented. Hence the lattice Γ is weakly supplemented but not supplemented. Since every nonzero element of Γ is essential in Γ, Γ is weakly essential supplemented but not essential supplemented.

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