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ON THE SOLVABILITY IN CLOSED-FORM OF A RATIONAL THREE-DIMENSIONAL SYSTEM OF DIFFERENCE EQUATIONS

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Abstract. In this paper, we are interested in the solvability in closed-form of the following three-dimensional system of nonlinear difference equations

$$x_{n+1} = \frac{a_1 x_{n-k} y_{n-k}}{a x_{n-k} + b y_{n-k} + c z_{n-k}},$$
$$y_{n+1} = \frac{a_2 y_{n-k} z_{n-k}}{a x_{n-k} + b y_{n-k} + c z_{n-k}},$$
$$z_{n+1} = \frac{a_3 x_{n-k} z_{n-k}}{a x_{n-k} + b y_{n-k} + c z_{n-k}},$$

where $n, k \in \mathbb{N}_0$, the parameters a_1, a_2, a_3, a, b, c are real numbers, and the initial values $x_{-k}, \ldots, x_0, y_{-k}, \ldots, y_0, z_{-k}, \ldots, z_0$, are non-zero real numbers. Firstly, we establish some preliminaries results for the general case, then we solve in a closed form, via some change of variables, our system in the two particular cases $k = 0$ and $k = 1$.

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1. Introduction and preliminary results

A huge number of real-world phenomena in various domains of sciences are described using different models of non-linear difference equations and systems. These models find applications in biology, where they describe population dynamics, disease spread, and predator-prey interactions; in economics, where they explain market fluctuations, economic growth, and financial forecasting; and in engineering, where they are used for control systems, feedback mechanisms, and signal processing. This practical significance has motivated researchers to work on this subject, as shown by the following contributions [3, 5, 7, 12, 13].

To provide a tool that allows to understand the behavior of the corresponding phenomena, many papers, investigated the solvability of some difference equations and

systems, for some examples of solvable models, we can consult the following references [1, 4, 6, 8–11].

In [2], Elsayed et al. derived explicit solution expressions for the following three dimensional system of difference equations

$$x_{n+1} = \frac{a_1 y_{n-1} z_{n-1}}{x_{n-1} + y_{n-1} + z_{n-1}},$$
$$y_{n+1} = \frac{a_2 x_{n-1} z_{n-1}}{x_{n-1} + y_{n-1} + z_{n-1}},$$
$$z_{n+1} = \frac{a_3 x_{n-1} y_{n-1}}{x_{n-1} + y_{n-1} + z_{n-1}}.$$

Among others, and as extension of the previous system, Zabat et al. in [14], solved in closed form the more general system

$$x_{n+1} = \frac{a_1 y_{n-1} z_{n-1}}{a x_{n-1} + b y_{n-1} + c z_{n-1}},$$
$$y_{n+1} = \frac{a_2 x_{n-1} z_{n-1}}{a x_{n-1} + b y_{n-1} + c z_{n-1}},$$
$$z_{n+1} = \frac{a_3 x_{n-1} y_{n-1}}{a x_{n-1} + b y_{n-1} + c z_{n-1}}.$$

As a contribution to this area of research, and motivated by our investigation in [14], we consider here the following rational three-dimensional system of difference equations defined by

$$\begin{aligned} x_{n+1} &= \frac{a_1 x_{n-k} y_{n-k}}{a x_{n-k} + b y_{n-k} + c z_{n-k}}, \\ y_{n+1} &= \frac{a_2 y_{n-k} z_{n-k}}{a x_{n-k} + b y_{n-k} + c z_{n-k}}, \\ z_{n+1} &= \frac{a_3 x_{n-k} z_{n-k}}{a x_{n-k} + b y_{n-k} + c z_{n-k}}, \end{aligned} \tag{1.1}$$

where $n, k \in \mathbb{N}_0$, the parameters a_1, a_2, a_3, a, b, c are real numbers, and the initial values $x_{-k}, \ldots, x_0, y_{-k}, \ldots, y_0, z_{-k}, \ldots, z_0$, are non-zero real numbers.

We will show that System (1.1) is transformable to a $6k+6$-periodic system of difference equations, and as consequence System (1.1) is reduced to three independents linear difference equations of order $k+1$. We will investigate the solvability of System (1.1) in details in the two cases $k = 0$ and $k = 1$. The interested readers can use the technique developed here to deal with the cases $k = 2, 3, \ldots$.

Definition 1. By a well defined-solution $(x_n, y_n, z_n)_{n \geq -k}$ of System (1.1), we mean a solution such that

$$a x_{n-k} + b y_{n-k} + c z_{n-k} \neq 0, \qquad n \in \mathbb{N}_0.$$

Lemma 1. *Let $(x_n, y_n, z_n)_{n \geq -k}$ be a well defined solution of System (1.1). Then,*

$$x_n . y_n . z_n \neq 0, \qquad n = -k, -k+1, \ldots.$$

Proof. Assume for example that there exists $n_0 \geq -k$ such that $x_{n_0} = 0$. It is not hard to see that $x_{n_0+k+1} = 0$ and $z_{n_0+k+1} = 0$, and consequently, we get $x_{n_0+2k+2} = 0, y_{n_0+2k+2} = 0$ and $z_{n_0+2k+2} = 0$. From these last equalities, it follows that $x_{n_0+3k+3} = \frac{0}{0}, y_{n_0+3k+3} = \frac{0}{0}$ and $z_{n_0+3k+3} = \frac{0}{0}$. That is the terms $x_{n_0+3k+3}, y_{n_0+3k+3}, z_{n_0+3k+3}$ are undefined. □

From here on, by a solution of (1.1), we mean a well-defined solution.

Lemma 2. *Using the following change of variables*

$$u_n = \frac{x_n}{y_n}, \quad v_n = \frac{y_n}{z_n}, \quad w_n = \frac{z_n}{x_n}, \quad n = -k, -k+1, \ldots, \tag{1.2}$$

System (1.1)*, is transformable to the following system*

$$u_{n+1} = \frac{a_1}{a_2 w_{n-k}}, \quad v_{n+1} = \frac{a_2}{a_3 u_{n-k}}, \quad w_{n+1} = \frac{a_3}{a_1 v_{n-k}}, \quad n = 0, 1, \ldots, \tag{1.3}$$

and, we get System (1.1) *reduced to the three independents linear difference equations*

$$x_{n+1} = \frac{a_1}{a u_{n-k} + c u_{n-k} w_{n-k} + b} x_{n-k}, \qquad n \in \mathbb{N}_0, \tag{1.4}$$

$$y_{n+1} = \frac{a_2}{a u_{n-k} v_{n-k} + b v_{n-k} + c} y_{n-k}, \qquad n \in \mathbb{N}_0, \tag{1.5}$$

$$z_{n+1} = \frac{a_3}{b v_{n-k} w_{n-k} + c w_{n-k} + a} z_{n-k}, \qquad n \in \mathbb{N}_0. \tag{1.6}$$

Proof. From (1.1), we get

$$\frac{x_{n+1}}{y_{n+1}} = \frac{a_1 x_{n-k}}{a_2 z_{n-k}}, \quad \frac{y_{n+1}}{z_{n+1}} = \frac{a_2 y_{n-k}}{a_3 x_{n-k}}, \quad \frac{z_{n+1}}{x_{n+1}} = \frac{a_3 z_{n-k}}{a_1 y_{n-k}},$$

so, by the change of variables (1.2), we get

$$u_{n+1} = \frac{a_1}{a_2 w_{n-k}}, \quad v_{n+1} = \frac{a_2}{a_3 u_{n-k}}, \quad w_{n+1} = \frac{a_3}{a_1 v_{n-k}}.$$

The formulae (1.4), (1.5) and (1.6), follows directly from (1.1) and (1.2). □

To solve (1.1), we need to solve System (1.3) and equations (1.4), (1.5) and (1.6). In the following result, we show that every solution of System (1.3) is periodic with period $6k+6$.

Lemma 3. *Let* $(u_n, v_n, w_n)_{n \geq -k}$ *be a solution of System* (1.3)*, then*

$$u_{n+6k+6} = u_n, \quad v_{n+6k+6} = v_n, \quad w_{n+6k+6} = w_n, \quad n = -k, -k+1, \ldots,$$

that is the solution is periodic of period $6k+6$*.*

Proof. From (1.3), we have

$$u_{n+6k+6} = \frac{a_1}{a_2 w_{n+5k+5}}, \quad v_{n+6k+6} = \frac{a_2}{a_3 u_{n+5k+5}}, \quad w_{n+6k+6} = \frac{a_3}{a_1 v_{n+5k+5}}, \tag{1.7}$$

$$u_{n+5k+5} = \frac{a_1}{a_2 w_{n+4k+4}}, \quad v_{n+5k+5} = \frac{a_2}{a_3 u_{n+4k+4}}, \quad w_{n+5k+5} = \frac{a_3}{a_1 v_{n+4k+4}}, \tag{1.8}$$

$$u_{n+4k+4}=\frac{a_1}{a_2w_{n+3k+3}},\quad v_{n+4k+4}=\frac{a_2}{a_3u_{n+3k+3}},\quad w_{n+4k+4}=\frac{a_3}{a_1v_{n+3k+3}},\tag{1.9}$$

$$u_{n+3k+3}=\frac{a_1}{a_2w_{n+2k+2}},\quad v_{n+3k+3}=\frac{a_2}{a_3u_{n+2k+2}},\quad w_{n+3k+3}=\frac{a_3}{a_1v_{n+2k+2}},\tag{1.10}$$

$$u_{n+2k+2}=\frac{a_1}{a_2w_{n+k+1}},\quad v_{n+2k+2}=\frac{a_2}{a_3u_{n+1k+1}},\quad w_{n+2k+2}=\frac{a_3}{a_1v_{n+k+1}},\tag{1.11}$$

$$u_{n+k+1}=\frac{a_1}{a_2w_n},\qquad v_{n+k+1}=\frac{a_2}{a_3u_n},\qquad w_{n+k+1}=\frac{a_3}{a_1v_n}.\tag{1.12}$$

From (1.7)-(1.12), we get

$$u_{n+6k+6}=\frac{a_1}{a_2w_{n+5k+5}}=\frac{a_1^2}{a_2a_3}v_{n+4k+4}=\frac{a_1^2}{a_3^2u_{n+3k+3}}=\frac{a_1a_2}{a_3^2}w_{n+2k+2}$$
$$=\frac{a_2}{a_3v_{n+k+1}}=u_n.$$

Similarly, we prove that

$$v_{n+6k+6}=v_n,\quad w_{n+6k+6}=w_n.$$

□

Lemma 4. *Let $(x_n,y_n,z_n)_{n\geq -k}$ be a solution of System (1.1). Assume that the parameters a_1,a_2,a_3,a,b,c and the initial values $x_{-k},\dots,x_0,y_{-k},\dots,y_0,z_{-k},\dots,z_0$ are positive and assume that $a_1<b,a_2<c,a_3<a$, then for every solution of (1.1), we have*

$$\lim_{n\to+\infty}(x_n,y_n,z_n)=(0,0,0).$$

Proof. Using the assumption on the parameters and the initial values, we get that the every solution of System (1.1) is positive, and it follows that

$$x_{n+1}\leq\frac{a_1}{b}x_{n-k},\quad y_{n+1}\leq\frac{a_2}{c}y_{n-k},\quad z_{n+1}\leq\frac{a_3}{a}z_{n-k},\quad n\in\mathbb{N}_0,$$

from which, we get for $i=1,2,\dots,k+1$, that

$$x_{(k+1)n+i}\leq\left(\frac{a_1}{b}\right)^{n+1}x_{i-k-1},$$
$$y_{(k+1)n+i}\leq\left(\frac{a_2}{c}\right)^{n+1}y_{i-k-1},$$
$$z_{(k+1)n+i}\leq\left(\frac{a_3}{a}\right)^{n+1}z_{i-k-1}.$$

Using the fact that $a_1<b,a_2<c,a_3<a$, the desired result follows. □

In the following sections, we solve System (1.1) when $k=0$ and $k=1$. To do this, we need in each case to solve the corresponding system (1.3) and the corresponding equations (1.4), (1.5) and (1.6).

2. THE CASE $k = 0$.

In this case the system (1.1) takes the form

$$\begin{aligned} x_{n+1} &= \frac{a_1 x_n y_n}{a x_n + b y_n + c z_n}, \\ y_{n+1} &= \frac{a_2 y_n z_n}{a x_n + b y_n + c z_n}, \\ z_{n+1} &= \frac{a_3 x_n z_n}{a x_n + b y_n + c z_n}. \end{aligned} \tag{2.1}$$

For System (2.1), the corresponding system (1.3), will be

$$u_{n+1} = \frac{a_1}{a_2 w_n}, \quad v_{n+1} = \frac{a_2}{a_3 u_n}, \quad w_{n+1} = \frac{a_3}{a_1 v_n}.$$

From Lemma (3), we get that the sequences $(u_n)_{n\geq 0}, (v_n)_{n\geq 0}$ and $(w_n)_{n\geq 0}$ are periodic of period 6 and we have

$$u_{6n} = u_0, \qquad u_{6n+1} = \frac{a_1}{a_2 w_0}, \qquad u_{6n+2} = \frac{a_1^2}{a_2 a_3} v_0, \tag{2.2}$$

$$u_{6n+3} = \frac{a_1^2}{a_3^2 u_0}, \qquad u_{6n+4} = \frac{a_1 a_2}{a_3^2} w_0, \qquad u_{6n+5} = \frac{a_2}{a_3 v_0}, \tag{2.3}$$

$$v_{6n} = v_0, \qquad v_{6n+1} = \frac{a_2}{a_3 u_0}, \qquad v_{6n+2} = \frac{a_2^2}{a_1 a_3} w_0, \tag{2.4}$$

$$v_{6n+3} = \frac{a_2^2}{a_1^2 v_0}, \qquad v_{6n+4} = \frac{a_2 a_3}{a_1^2} u_0, \qquad v_{6n+5} = \frac{a_3}{a_1 w_0}, \tag{2.5}$$

$$w_{6n} = w_0, \qquad w_{6n+1} = \frac{a_3}{a_1 v_0}, \qquad w_{6n+2} = \frac{a_3^2}{a_1 a_2} u_0, \tag{2.6}$$

$$w_{6n+3} = \frac{a_3^2}{a_2^2 w_0}, \qquad w_{6n+4} = \frac{a_1 a_3}{a_2^2} v_0, \qquad w_{6n+5} = \frac{a_1}{a_2 u_0}. \tag{2.7}$$

Also for System (2.1), the corresponding equations (1.4), (1.5) and (1.6) will be

$$x_{n+1} = \frac{a_1}{a u_n + c u_n w_n + b} x_n, \qquad n \in \mathbb{N}_0, \tag{2.8}$$

$$y_{n+1} = \frac{a_2}{a u_n v_n + b v_n + c} y_n, \qquad n \in \mathbb{N}_0, \tag{2.9}$$

$$z_{n+1} = \frac{a_3}{b v_n w_n + c w_n + a} z_n, \qquad n \in \mathbb{N}_0. \tag{2.10}$$

Now, from (1.2), (2.2)-(2.7) and (2.8)-(2.10), we get

$$x_{6n+1} = \alpha_0 x_{6n}, \tag{2.11}$$

$$x_{6n+2} = \alpha_1 \alpha_0 x_{6n}, \tag{2.12}$$

$$x_{6n+3} = \alpha_2 \alpha_1 \alpha_0 x_{6n}, , \tag{2.13}$$

$$x_{6n+4} = \alpha_3\alpha_2\alpha_1\alpha_0 x_{6n}, \tag{2.14}$$
$$x_{6n+5} = \alpha_4\alpha_3\alpha_2\alpha_1\alpha_0 x_{6n}, \tag{2.15}$$
$$x_{6n+6} = \alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0 x_{6n}, \tag{2.16}$$

$$y_{6n+1} = \beta_0 y_{6n}, \tag{2.17}$$
$$y_{6n+2} = \beta_1\beta_0 y_{6n}, \tag{2.18}$$
$$y_{6n+3} = \beta_2\beta_1\beta_0 y_{6n}, \tag{2.19}$$
$$y_{6n+4} = \beta_3\beta_2\beta_1\beta_0 y_{6n}, \tag{2.20}$$
$$y_{6n+5} = \beta_4\beta_3\beta_2\beta_1\beta_0 y_{6n}, \tag{2.21}$$
$$y_{6n+6} = \beta_5\beta_4\beta_3\beta_2\beta_1\beta_0 y_{6n}, \tag{2.22}$$

$$z_{6n+1} = \gamma_0 z_{6n}, \tag{2.23}$$
$$z_{6n+2} = \gamma_1\gamma_0 z_{6n}, \tag{2.24}$$
$$z_{6n+3} = \gamma_2\gamma_1\gamma_0 z_{6n}, \tag{2.25}$$
$$z_{6n+4} = \gamma_3\gamma_2\gamma_1\gamma_0 z_{6n}, \tag{2.26}$$
$$z_{6n+5} = \gamma_4\gamma_3\gamma_2\gamma_1\gamma_0 z_{6n}, \tag{2.27}$$
$$z_{6n+6} = \gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0 z_{6n}, \tag{2.28}$$

where

$$\alpha_0 = \frac{a_1 y_0}{ax_0 + by_0 + cz_0}, \qquad \alpha_1 = \frac{a_1 a_2 y_0 z_0}{aa_1 x_0 y_0 + ba_2 y_0 z_0 + ca_3 x_0 z_0},$$
$$\alpha_2 = \frac{a_1 a_2^2 a_3 z_0}{aa_1^2 a_2 y_0 + ba_2^2 a_3 z_0 + ca_1 a_3^2 x_0}, \qquad \alpha_3 = \frac{a_1 a_2^2 a_3^2 x_0 z_0}{aa_1^2 a_2^2 y_0 z_0 + ba_2^2 a_3^2 x_0 z_0 + ca_1^2 a_3^2 x_0 y_0},$$
$$\alpha_4 = \frac{a_1 a_2 a_3^2 x_0}{aa_1 a_2^2 z_0 + ba_2 a_3^2 x_0 + ca_1^2 a_3 y_0}, \qquad \alpha_5 = \frac{a_1 a_3 x_0 y_0}{aa_2 x_0 z_0 + ba_3 x_0 y_0 + ca_1 y_0 z_0},$$
$$\beta_0 = \frac{a_2 z_0}{ax_0 + by_0 + cz_0}, \qquad \beta_1 = \frac{a_2 a_3 x_0 z_0}{aa_1 x_0 y_0 + ba_2 y_0 z_0 + ca_3 x_0 z_0},$$
$$\beta_2 = \frac{a_1 a_2 a_3^2 x_0}{aa_1^2 a_2 y_0 + ba_2^2 a_3 z_0 + ca_1 a_3^2 x_0}, \qquad \beta_3 = \frac{a_1^2 a_2 a_3^2 x_0 y_0}{aa_1^2 a_2^2 y_0 z_0 + ba_2^2 a_3^2 x_0 z_0 + ca_1^2 a_3^2 x_0 y_0},$$
$$\beta_4 = \frac{a_1^2 a_2 a_3 y_0}{aa_1 a_2^2 z_0 + ba_2 a_3^2 x_0 + ca_1^2 a_3 y_0}, \qquad \beta_5 = \frac{a_1 a_2 y_0 z_0}{aa_2 x_0 z_0 + ba_3 x_0 y_0 + ca_1 y_0 z_0},$$
$$\gamma_0 = \frac{a_3 x_0}{ax_0 + by_0 + cz_0}, \qquad \gamma_1 = \frac{a_1 a_3 x_0 y_0}{aa_1 x_0 y_0 + ba_2 y_0 z_0 + ca_3 x_0 z_0},$$
$$\gamma_2 = \frac{a_1^2 a_2 a_3 y_0}{aa_1^2 a_2 y_0 + ba_2^2 a_3 z_0 + ca_1 a_3^2 x_0}, \qquad \gamma_3 = \frac{a_1^2 a_2^2 a_3 y_0 z_0}{aa_1^2 a_2^2 y_0 z_0 + ba_2^2 a_3^2 x_0 z_0 + ca_1^2 a_3^2 x_0 y_0},$$
$$\gamma_4 = \frac{a_1 a_2^2 a_3 z_0}{aa_1 a_2^2 z_0 + ba_2 a_3^2 x_0 + ca_1^2 a_3 y_0}, \qquad \gamma_5 = \frac{a_2 a_3 x_0 z_0}{aa_2 x_0 z_0 + ba_3 x_0 y_0 + ca_1 y_0 z_0}.$$

Now, we will solve equations (2.11)-(2.28). We have $(x_{6n})_{n\geq 0}$ is the solution of the first-order linear homogenous difference equation given in (2.16), from which

$$x_{6n} = (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0. \tag{2.29}$$

Hence, from (2.29) and (2.11)-(2.15) it follows that

$$\begin{aligned}
x_{6n+1} &= \alpha_0 (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,\\
x_{6n+2} &= \alpha_1\alpha_0 (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,\\
x_{6n+3} &= \alpha_2\alpha_1\alpha_0 (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,\\
x_{6n+4} &= \alpha_3\alpha_2\alpha_1\alpha_0 (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,\\
x_{6n+5} &= \alpha_4\alpha_3\alpha_2\alpha_1\alpha_0 (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0.
\end{aligned}$$

Similarly, $(y_{6n})_{n\geq 0}$ is the solution of the first-order linear homogenous difference equation (2.22), and we have

$$y_{6n} = (\beta_0\beta_1\beta_2\beta_3\beta_4\beta_5)^n y_0. \tag{2.30}$$

Thus, from (2.30) and (2.17)-(2.21), we get

$$\begin{aligned}
y_{6n+1} &= \beta_0 (\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,\\
y_{6n+2} &= \beta_1\beta_0 (\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,\\
y_{6n+3} &= \beta_2\beta_1\beta_0 (\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,\\
y_{6n+4} &= \beta_3\beta_2\beta_1\beta_0 (\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,\\
y_{6n+5} &= \beta_4\beta_3\beta_2\beta_1\beta_0 (\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0.
\end{aligned}$$

Also, $(z_{6n})_{n\geq 0}$ is the solution of the first-order linear homogenous difference equation (2.28), and we have

$$z_{6n} = (\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5)^n z_0 \tag{2.31}$$

From (2.31) and (2.23)-(2.27), we obtain

$$\begin{aligned}
z_{6n+1} &= \gamma_0 (\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,\\
z_{6n+2} &= \gamma_1\gamma_0 (\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,\\
z_{6n+3} &= \gamma_2\gamma_1\gamma_0 (\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,\\
z_{6n+4} &= \gamma_3\gamma_2\gamma_1\gamma_0 (\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,\\
z_{6n+5} &= \gamma_4\gamma_3\gamma_2\gamma_1\gamma_0 (\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0.
\end{aligned}$$

In summary, the formulas of well-defined solutions of System (2.1) are presented in the following result.

Theorem 1. *Consider System (2.1) and let $(x_n, y_n, z_n)_{n\geq 0}$ be a solution of (2.1). Then, for $n = 0, 1, \ldots$, we have*

$$x_{6n} = (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0, \qquad x_{6n+1} = \alpha_0(\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,$$
$$x_{6n+2} = \alpha_1\alpha_0(\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0, \qquad x_{6n+3} = \alpha_2\alpha_1\alpha_0(\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,$$
$$x_{6n+4} = \alpha_3\alpha_2\alpha_1\alpha_0(\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0, \qquad x_{6n+5} = \alpha_4\alpha_3\alpha_2\alpha_1\alpha_0 \times (\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0)^n x_0,$$

$$y_{6n} = (\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0, \qquad y_{6n+1} = \beta_0(\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,$$
$$y_{6n+2} = \beta_1\beta_0(\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0, \qquad y_{6n+3} = \beta_2\beta_1\beta_0(\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,$$
$$y_{6n+4} = \beta_3\beta_2\beta_1\beta_0(\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0, \qquad y_{6n+5} = \beta_4\beta_3\beta_2\beta_1\beta_0(\beta_5\beta_4\beta_3\beta_2\beta_1\beta_0)^n y_0,$$

$$z_{6n} = (\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0, \qquad z_{6n+1} = \gamma_0(\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,$$
$$z_{6n+2} = \gamma_1\gamma_0(\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0, \qquad z_{6n+3} = \gamma_2\gamma_1\gamma_0(\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,$$
$$z_{6n+4} = \gamma_3\gamma_2\gamma_1\gamma_0(\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0, \qquad z_{6n+5} = \gamma_4\gamma_3\gamma_2\gamma_1\gamma_0(\gamma_5\gamma_4\gamma_3\gamma_2\gamma_1\gamma_0)^n z_0,$$

where

$$\alpha_0 = \frac{a_1 y_0}{ax_0 + by_0 + cz_0}, \qquad \alpha_1 = \frac{a_1 a_2 y_0 z_0}{aa_1 x_0 y_0 + ba_2 y_0 z_0 + ca_3 x_0 z_0},$$
$$\alpha_2 = \frac{a_1 a_2^2 a_3 z_0}{aa_1^2 a_2 y_0 + ba_2^2 a_3 z_0 + ca_1 a_3^2 x_0}, \qquad \alpha_3 = \frac{a_1 a_2^2 a_3^2 x_0 z_0}{aa_1^2 a_2^2 y_0 z_0 + ba_2^2 a_3^2 x_0 z_0 + ca_1^2 a_3^2 x_0 y_0},$$
$$\alpha_4 = \frac{a_1 a_2 a_3^2 x_0}{aa_1 a_2^2 z_0 + ba_2 a_3^2 x_0 + ca_1^2 a_3 y_0}, \qquad \alpha_5 = \frac{a_1 a_3 x_0 y_0}{aa_2 x_0 z_0 + ba_3 x_0 y_0 + ca_1 y_0 z_0},$$
$$\beta_0 = \frac{a_2 z_0}{ax_0 + by_0 + cz_0}, \qquad \beta_1 = \frac{a_2 a_3 x_0 z_0}{aa_1 x_0 y_0 + ba_2 y_0 z_0 + ca_3 x_0 z_0},$$
$$\beta_2 = \frac{a_1 a_2 a_3^2 x_0}{aa_1^2 a_2 y_0 + ba_2^2 a_3 z_0 + ca_1 a_3^2 x_0}, \qquad \beta_3 = \frac{a_1^2 a_2 a_3^2 x_0 y_0}{aa_1^2 a_2^2 y_0 z_0 + ba_2^2 a_3^2 x_0 z_0 + ca_1^2 a_3^2 x_0 y_0},$$
$$\beta_4 = \frac{a_1^2 a_2 a_3 y_0}{aa_1 a_2^2 z_0 + ba_2 a_3^2 x_0 + ca_1^2 a_3 y_0}, \qquad \beta_5 = \frac{a_1 a_2 y_0 z_0}{aa_2 x_0 z_0 + ba_3 x_0 y_0 + ca_1 y_0 z_0},$$
$$\gamma_0 = \frac{a_3 x_0}{ax_0 + by_0 + cz_0}, \qquad \gamma_1 = \frac{a_1 a_3 x_0 y_0}{aa_1 x_0 y_0 + ba_2 y_0 z_0 + ca_3 x_0 z_0},$$
$$\gamma_2 = \frac{a_1^2 a_2 a_3 y_0}{aa_1^2 a_2 y_0 + ba_2^2 a_3 z_0 + ca_1 a_3^2 x_0}, \qquad \gamma_3 = \frac{a_1^2 a_2^2 a_3 y_0 z_0}{aa_1^2 a_2^2 y_0 z_0 + ba_2^2 a_3^2 x_0 z_0 + ca_1^2 a_3^2 x_0 y_0},$$
$$\gamma_4 = \frac{a_1 a_2^2 a_3 z_0}{aa_1 a_2^2 z_0 + ba_2 a_3^2 x_0 + ca_1^2 a_3 y_0}, \qquad \gamma_5 = \frac{a_2 a_3 x_0 z_0}{aa_2 x_0 z_0 + ba_3 x_0 y_0 + ca_1 y_0 z_0}.$$

3. THE CASE $k = 1$

When $k = 1$, System (1.1) takes the form

$$\begin{aligned} x_{n+1} &= \frac{a_1 x_{n-1} y_{n-1}}{a x_{n-1} + b y_{n-1} + c z_{n-1}}, \\ y_{n+1} &= \frac{a_2 y_{n-1} z_{n-1}}{a x_{n-1} + b y_{n-1} + c z_{n-1}}, \\ z_{n+1} &= \frac{a_3 x_{n-1} z_{n-1}}{a x_{n-1} + b y_{n-1} + c z_{n-1}}, \end{aligned} \tag{3.1}$$

and the corresponding system (1.3) will be

$$u_{n+1} = \frac{a_1}{a_2 w_{n-1}}, \quad v_{n+1} = \frac{a_2}{a_3 u_{n-1}}, \quad w_{n+1} = \frac{a_3}{a_1 v_{n-1}}.$$

From Lemma 3, it follows that the sequences $(u_n)_{n\geq -1}, (v_n)_{n\geq -1}$ and $(w_n)_{n\geq -1}$ are periodic of period 12 and we have

$$u_{12n-1} = u_{-1}, \qquad u_{12n} = u_0, \qquad u_{12n+1} = \frac{a_1}{a_2 w_{-1}}, \tag{3.2}$$

$$u_{12n+2} = \frac{a_1}{a_2 w_0}, \qquad u_{12n+3} = \frac{a_1^2}{a_2 a_3} v_{-1}, \qquad u_{12n+4} = \frac{a_1^2}{a_2 a_3} v_0, \tag{3.3}$$

$$u_{12n+5} = \frac{a_1^2}{a_3^2 u_{-1}}, \qquad u_{12n+6} = \frac{a_1^2}{a_3^2 u_0}, \qquad u_{12n+7} = \frac{a_1 a_2}{a_3^2} w_{-1}, \tag{3.4}$$

$$u_{12n+8} = \frac{a_1 a_2}{a_3^2} w_0, \qquad u_{12n+9} = \frac{a_2}{a_3 v_{-1}}, \qquad u_{12n+10} = \frac{a_2}{a_3 v_0}, \tag{3.5}$$

$$v_{12n-1} = v_{-1}, \qquad v_{12n} = v_0, \qquad v_{12n+1} = \frac{a_2}{a_3 u_{-1}}, \tag{3.6}$$

$$v_{12n+2} = \frac{a_2}{a_3 u_0}, \qquad v_{12n+3} = \frac{a_2^2}{a_1 a_3} w_{-1}, \qquad v_{12n+4} = \frac{a_2^2}{a_1 a_3} w_0, \tag{3.7}$$

$$v_{12n+5} = \frac{a_2^2}{a_1^2 v_{-1}}, \qquad v_{12n+6} = \frac{a_2^2}{a_1^2 v_0}, \qquad v_{12n+7} = \frac{a_2 a_3}{a_1^2} w_{-1}, \tag{3.8}$$

$$v_{12n+8} = \frac{a_2 a_3}{a_1^2} w_0, \qquad v_{12n+9} = \frac{a_3}{a_1 w_{-1}}, \qquad v_{12n+10} = \frac{a_3}{a_1 w_0}, \tag{3.9}$$

$$w_{12n-1} = w_{-1}, \qquad w_{12n} = w_0, \qquad w_{12n+1} = \frac{a_3}{a_1 v_{-1}}, \tag{3.10}$$

$$w_{12n+2} = \frac{a_3}{a_1 v_0}, \qquad w_{12n+3} = \frac{a_3^2}{a_1 a_2} w_{-1}, \qquad w_{12n+4} = \frac{a_3^2}{a_1 a_2} u_0, \tag{3.11}$$

$$w_{12n+5} = \frac{a_3^2}{a_2^2 w_{-1}}, \qquad w_{12n+6} = \frac{a_3^2}{a_2^2 w_0}, \qquad w_{12n+7} = \frac{a_1 a_3}{a_2^2} v_{-1}, \tag{3.12}$$

$$w_{12n+8} = \frac{a_1 a_3}{a_2^2} v_0, \qquad w_{12n+9} = \frac{a_1}{a_2 w_{-1}}, \qquad w_{12n+10} = \frac{a_1}{a_2 w_0}. \tag{3.13}$$

For system (3.1), the corresponding equations (1.4)-(1.6) will be

$$x_{n+1} = \frac{a_1}{au_{n-1} + cu_{n-1}w_{n-1} + b} x_{n-1}, \qquad n \in \mathbb{N}_0, \tag{3.14}$$

$$y_{n+1} = \frac{a_2}{au_{n-1}v_{n-1} + bv_{n-1} + c} y_{n-1}, \qquad n \in \mathbb{N}_0, \tag{3.15}$$

$$z_{n+1} = \frac{a_3}{bv_{n-1}w_{n-1} + cw_{n-1} + a} z_{n-1}, \qquad n \in \mathbb{N}_0. \tag{3.16}$$

Now, from (1.2), (3.2)-(3.13) and the equations (3.14)-(3.16), we obtain

$$x_{12n+1} = \alpha_0 x_{12n-1}, \tag{3.17}$$
$$x_{12n+2} = \alpha_1 x_{12n}, \tag{3.18}$$
$$x_{12n+3} = \alpha_0\alpha_2 x_{12n-1}, \tag{3.19}$$
$$x_{12n+4} = \alpha_1\alpha_3 x_{12n}, \tag{3.20}$$
$$x_{12n+5} = \alpha_0\alpha_2\alpha_4 x_{12n-1}, \tag{3.21}$$
$$x_{12n+6} = \alpha_1\alpha_3\alpha_5 x_{12n}, \tag{3.22}$$
$$x_{12n+7} = \alpha_0\alpha_2\alpha_4\alpha_6 x_{12n-1}, \tag{3.23}$$
$$x_{12n+8} = \alpha_1\alpha_3\alpha_5\alpha_7 x_{12n}, \tag{3.24}$$
$$x_{12n+9} = \alpha_0\alpha_2\alpha_4\alpha_6\alpha_8 x_{12n-1}, \tag{3.25}$$
$$x_{12n+10} = \alpha_1\alpha_3\alpha_5\alpha_7\alpha_9 x_{12n}, \tag{3.26}$$
$$x_{12n+11} = \alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10} x_{12n-1}, \tag{3.27}$$
$$x_{12n+12} = \alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11} x_{12n}, \tag{3.28}$$

$$y_{12n+1} = \beta_0 y_{12n-1}, \tag{3.29}$$
$$y_{12n+2} = \beta_1 y_{12n}, \tag{3.30}$$
$$y_{12n+3} = \beta_0\beta_2 y_{12n-1}, \tag{3.31}$$
$$y_{12n+4} = \beta_3\beta_1 y_{12n}, \tag{3.32}$$
$$y_{12n+5} = \beta_0\beta_2\beta_4 y_{12n-1}, \tag{3.33}$$
$$y_{12n+6} = \beta_1\beta_3\beta_5 y_{12n}, \tag{3.34}$$
$$y_{12n+7} = \beta_0\beta_2\beta_4\beta_6 y_{12n-1}, \tag{3.35}$$
$$y_{12n+8} = \beta_1\beta_3\beta_5\beta_7 y_{12n}, \tag{3.36}$$
$$y_{12n+9} = \beta_0\beta_2\beta_4\beta_6\beta_8 y_{12n-1}, \tag{3.37}$$
$$y_{12n+10} = \beta_1\beta_3\beta_5\beta_7\beta_9 y_{12n}, \tag{3.38}$$
$$y_{12n+11} = \beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10} y_{12n-1}, \tag{3.39}$$
$$y_{12n+12} = \beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11} y_{12n}, \tag{3.40}$$

$$z_{12n+1} = \gamma_0 z_{12n-1}, \tag{3.41}$$
$$z_{12n+2} = \gamma_1 z_{12n}, \tag{3.42}$$
$$z_{12n+3} = \gamma_0\gamma_2 z_{12n-1}, \tag{3.43}$$
$$z_{12n+4} = \gamma_3\gamma_1 z_{12n}, \tag{3.44}$$
$$z_{12n+5} = \gamma_4\gamma_2\gamma_0 z_{12n-1}, \tag{3.45}$$
$$z_{12n+6} = \gamma_5\gamma_3\gamma_1 z_{12n}, \tag{3.46}$$
$$z_{12n+7} = \gamma_0\gamma_2\gamma_4\gamma_6 z_{12n-1}, \tag{3.47}$$
$$z_{12n+8} = \gamma_1\gamma_3\gamma_5\gamma_7 z_{12n}, \tag{3.48}$$
$$z_{12n+9} = \gamma_0\gamma_2\gamma_4\gamma_6\gamma_8 z_{12n-1}, \tag{3.49}$$
$$z_{12n+10} = \gamma_1\gamma_3\gamma_5\gamma_7\gamma_9 z_{12n}, \tag{3.50}$$
$$z_{12n+11} = \gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10} z_{12n-1}, \tag{3.51}$$
$$z_{12n+12} = \gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11} z_{12n}, \tag{3.52}$$

where

$$\alpha_0 = \frac{a_1 y_{-1}}{a x_{-1} + b y_{-1} + c z_{-1}},$$
$$\alpha_1 = \frac{a_1 y_0}{a x_0 + b y_0 + c z_0},$$
$$\alpha_2 = \frac{a_1 a_2 y_{-1} z_{-1}}{a a_1 x_{-1} y_{-1} + b a_2 y_{-1} z_{-1} + c a_3 x_{-1} z_{-1}},$$
$$\alpha_3 = \frac{a_1 a_2 y_0 z_0}{a a_1 x_0 y_0 + b a_2 y_0 z_0 + c a_3 x_0 z_0},$$
$$\alpha_4 = \frac{a_1 a_2^2 a_3 z_{-1}}{a a_1^2 a_2 y_{-1} + b a_2^2 a_3 z_{-1} + c a_1 a_3^2 x_{-1}},$$
$$\alpha_5 = \frac{a_1 a_2^2 a_3 z_0}{a a_1^2 a_2 y_0 + b a_2^2 a_3 z_0 + c a_1 a_3^2 x_0},$$
$$\alpha_6 = \frac{a_1 a_2^2 a_3^2 x_{-1} z_{-1}}{a a_1^2 a_2^2 y_{-1} z_{-1} + b a_2^2 a_3^2 x_{-1} z_{-1} + c a_1^2 a_3^2 x_{-1} y_{-1}},$$
$$\alpha_7 = \frac{a_1 a_2^2 a_3^2 x_0 z_0}{a a_1^2 a_2^2 y_0 z_0 + b a_2^2 a_3^2 x_0 z_0 + c a_1^2 a_3^2 x_0 y_0},$$
$$\alpha_8 = \frac{a_1 a_2 a_3^2 x_{-1}}{a a_1 a_2^2 z_{-1} + b a_2 a_3^2 x_{-1} + c a_1^2 a_3 y_{-1}},$$
$$\alpha_9 = \frac{a_1 a_2 a_3^2 x_0}{a a_1 a_2^2 z_0 + b a_2 a_3^2 x_0 + c a_1^2 a_3 y_0},$$
$$\alpha_{10} = \frac{a_1 a_3 x_{-1} y_{-1}}{a a_2 x_{-1} z_{-1} + b a_3 x_{-1} y_{-1} + c a_1 y_{-1} z_{-1}},$$

$$\alpha_{11} = \frac{a_1 a_3 x_0 y_0}{a a_2 x_0 z_0 + b a_3 x_0 y_0 + c a_1 y_0 z_0},$$

$$\beta_0 = \frac{a_2 z_{-1}}{a x_{-1} + b y_{-1} + c z_{-1}},$$
$$\beta_1 = \frac{a_2 z_0}{a x_0 + b y_0 + c z_0},$$
$$\beta_2 = \frac{a_2 a_3 x_{-1} z_{-1}}{a a_1 x_{-1} y_{-1} + b a_2 y_{-1} z_{-1} + c a_3 x_{-1} z_{-1}},$$
$$\beta_3 = \frac{a_2 a_3 x_0 z_0}{a a_1 x_0 y_0 + b a_2 y_0 z_0 + c a_3 x_0 z_0},$$
$$\beta_4 = \frac{a_1 a_2 a_3^2 x_{-1}}{a a_1^2 a_2 y_{-1} + b a_2^2 a_3 z_{-1} + c a_1 a_3^2 x_{-1}},$$
$$\beta_5 = \frac{a_1 a_2 a_3^2 x_0}{a a_1^2 a_2 y_0 + b a_2^2 a_3 z_0 + c a_1 a_3^2 x_0},$$
$$\beta_6 = \frac{a_1^2 a_2 a_3^2 x_{-1} y_{-1}}{a a_1^2 a_2^2 y_{-1} z_{-1} + b a_2^2 a_3^2 x_{-1} z_{-1} + c a_1^2 a_3^2 x_{-1} y_{-1}},$$
$$\beta_7 = \frac{a_1^2 a_2 a_3^2 x_0 y_0}{a a_1^2 a_2^2 y_0 z_0 + b a_2^2 a_3^2 x_0 z_0 + c a_1^2 a_3^2 x_0 y_0},$$
$$\beta_8 = \frac{a_1^2 a_2 a_3 y_{-1}}{a a_1 a_2^2 z_{-1} + b a_2 a_3^2 x_{-1} + c a_1^2 a_3 y_{-1}},$$
$$\beta_9 = \frac{a_1^2 a_2 a_3 y_0}{a a_1 a_2^2 z_0 + b a_2 a_3^2 x_0 + c a_1^2 a_3 y_0},$$
$$\beta_{10} = \frac{a_1 a_2 y_{-1} z_{-1}}{a a_2 x_{-1} z_{-1} + b a_3 x_{-1} y_{-1} + c a_1 y_{-1} z_{-1}},$$
$$\beta_{11} = \frac{a_1 a_2 y_0 z_0}{a a_2 x_0 z_0 + b a_3 x_0 y_0 + c a_1 y_0 z_0},$$

$$\gamma_0 = \frac{a_3 x_{-1}}{a x_{-1} + b y_{-1} + c z_{-1}},$$
$$\gamma_1 = \frac{a_3 x_0}{a x_0 + b y_0 + c z_0},$$
$$\gamma_2 = \frac{a_1 a_3 x_{-1} y_{-1}}{a a_1 x_{-1} y_{-1} + b a_2 y_{-1} z_{-1} + c a_3 x_{-1} z_{-1}},$$
$$\gamma_3 = \frac{a_1 a_3 x_0 y_0}{a a_1 x_0 y_0 + b a_2 y_0 z_0 + c a_3 x_0 z_0},$$
$$\gamma_4 = \frac{a_1^2 a_2 a_3 y_{-1}}{a a_1^2 a_2 y_{-1} + b a_2^2 a_3 z_{-1} + c a_1 a_3^2 x_{-1}},$$

$$\gamma_5 = \frac{a_1^2 a_2 a_3 y_0}{a a_1^2 a_2 y_0 + b a_2^2 a_3 z_0 + c a_1 a_3^2 x_0},$$
$$\gamma_6 = \frac{a_1^2 a_2^2 a_3 y_{-1} z_{-1}}{a a_1^2 a_2^2 y_{-1} z_{-1} + b a_2^2 a_3^2 x_{-1} z_{-1} + c a_1^2 a_3^2 x_{-1} y_{-1}},$$
$$\gamma_7 = \frac{a_1^2 a_2^2 a_3 y_0 z_0}{a a_1^2 a_2^2 y_0 z_0 + b a_2^2 a_3^2 x_0 z_0 + c a_1^2 a_3^2 x_0 y_0},$$
$$\gamma_8 = \frac{a_1 a_2^2 a_3 z_{-1}}{a a_1 a_2^2 z_{-1} + b a_2 a_3^2 x_{-1} + c a_1^2 a_3 y_{-1}},$$
$$\gamma_9 = \frac{a_1 a_2^2 a_3 z_0}{a a_1 a_2^2 z_0 + b a_2 a_3^2 x_0 + c a_1^2 a_3 y_0},$$
$$\gamma_{10} = \frac{a_2 a_3 x_{-1} z_{-1}}{a a_2 x_{-1} z_{-1} + b a_3 x_{-1} y_{-1} + c a_1 y_{-1} z_{-1}},$$
$$\gamma_{11} = \frac{a_2 a_3 z_0 x_0}{a a_2 x_0 z_0 + b a_3 x_0 y_0 + c a_1 y_0 z_0}.$$

Now, we will solve Equations (3.17)-(3.28). The equation (3.28), satisfies a first order homogenous difference equation, and its general solution is given by

$$x_{12n} = (\alpha_1 \alpha_3 \alpha_5 \alpha_7 \alpha_9 \alpha_{11})^n x_0, \tag{3.53}$$

The equation (3.27), can be written as

$$x_{12n+11} = \alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10} x_{12(n-1)+11},$$

which is also a first-order homogenous linear difference equation, and its general solution is

$$x_{12n-1} = (\alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10})^n x_{-1}. \tag{3.54}$$

It follows from (3.53) and (3.54) and the equations (3.17)-(3.27), that we have for all $n \in \mathbb{N}_0$:

$$\begin{aligned}
x_{12n-1} &= (\alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10})^n x_{-1}, \\
x_{12n} &= (\alpha_1 \alpha_3 \alpha_5 \alpha_7 \alpha_9 \alpha_{11})^n x_0, \\
x_{12n+1} &= \alpha_0 (\alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10})^n x_{-1}, \\
x_{12n+2} &= \alpha_1 (\alpha_1 \alpha_3 \alpha_5 \alpha_7 \alpha_9 \alpha_{11})^n x_0, \\
x_{12n+3} &= \alpha_0 \alpha_2 (\alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10})^n x_{-1}, \\
x_{12n+4} &= \alpha_1 \alpha_3 (\alpha_1 \alpha_3 \alpha_5 \alpha_7 \alpha_9 \alpha_{11})^n x_0, \\
x_{12n+5} &= \alpha_0 \alpha_2 \alpha_4 (\alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10})^n x_{-1}, \\
x_{12n+6} &= \alpha_1 \alpha_3 \alpha_5 (\alpha_1 \alpha_3 \alpha_5 \alpha_7 \alpha_9 \alpha_{11})^n x_0, \\
x_{12n+7} &= \alpha_0 \alpha_2 \alpha_4 \alpha_6 (\alpha_0 \alpha_2 \alpha_4 \alpha_6 \alpha_8 \alpha_{10})^n x_{-1}, \\
x_{12n+8} &= \alpha_1 \alpha_3 \alpha_5 \alpha_7 (\alpha_1 \alpha_3 \alpha_5 \alpha_7 \alpha_9 \alpha_{11})^n x_0,
\end{aligned}$$

$$x_{12n+9} = \alpha_0\alpha_2\alpha_4\alpha_6\alpha_8(\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},$$
$$x_{12n+10} = \alpha_1\alpha_3\alpha_5\alpha_7\alpha_9(\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,$$

Now, we move to the equations (3.29)-(3.40). Similarly, (3.40) and (3.39) are first-order linear homogeneous difference equations and their general solutions are as follows

$$y_{12n-1} = (\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1}, \tag{3.55}$$
$$y_{12n} = (\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0, \tag{3.56}$$

From (3.55) and (3.56) and the equations (3.29)-(3.38), we get that

$$y_{12n+1} = \beta_0(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},$$
$$y_{12n+2} = \beta_1(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,$$
$$y_{12n+3} = \beta_0\beta_2(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},$$
$$y_{12n+4} = \beta_1\beta_3(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,$$
$$y_{12n+5} = \beta_0\beta_2\beta_4(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},$$
$$y_{12n+6} = \beta_1\beta_3\beta_5(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,$$
$$y_{12n+7} = \beta_0\beta_2\beta_4\beta_6(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},$$
$$y_{12n+8} = \beta_1\beta_3\beta_5\beta_7(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,$$
$$y_{12n+9} = \beta_0\beta_2\beta_4\beta_6\beta_8(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},$$
$$y_{12n+10} = \beta_1\beta_3\beta_5\beta_7\beta_9(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,$$

To conclude, we solve the equations (3.41) and (3.52). We begin with the first-order linear homogeneous difference equations (3.51) and (3.52). We have,

$$z_{12n-1} = (\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1}, \tag{3.57}$$
$$z_{12n} = (\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0, \tag{3.58}$$

From (3.57) and (3.58), and the equations (3.41)-(3.50), we get

$$z_{12n+1} = \gamma_0(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},$$
$$z_{12n+2} = \gamma_1(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,$$
$$z_{12n+3} = \gamma_0\gamma_2(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},$$
$$z_{12n+4} = \gamma_1\gamma_3(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,$$
$$z_{12n+5} = \gamma_0\gamma_2\gamma_4(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},$$
$$z_{12n+6} = \gamma_1\gamma_3\gamma_5(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,$$
$$z_{12n+7} = \gamma_0\gamma_2\gamma_4\gamma_6(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},$$
$$z_{12n+8} = \gamma_1\gamma_3\gamma_5\gamma_7(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,$$
$$z_{12n+9} = \gamma_0\gamma_2\gamma_4\gamma_6\gamma_8(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},$$

$$z_{12n+10} = \gamma_1\gamma_3\gamma_5\gamma_7\gamma_9(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,$$

In summary, we have the following result.

Theorem 2. *Let $(x_n, y_n, z_n)_{n\geq -1}$ be a solution of System (3.1), then for $n \geq 0$, we have*

$$\begin{aligned}
x_{12n-1} &= (\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},\\
x_{12n} &= (\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,\\
x_{12n+1} &= \alpha_0(\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},\\
x_{12n+2} &= \alpha_1(\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,\\
x_{12n+3} &= \alpha_0\alpha_2(\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},\\
x_{12n+4} &= \alpha_1\alpha_3(\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,\\
x_{12n+5} &= \alpha_0\alpha_2\alpha_4(\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},\\
x_{12n+6} &= \alpha_1\alpha_3\alpha_5(\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,\\
x_{12n+7} &= \alpha_0\alpha_2\alpha_4\alpha_6(\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},\\
x_{12n+8} &= \alpha_1\alpha_3\alpha_5\alpha_7(\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,\\
x_{12n+9} &= \alpha_0\alpha_2\alpha_4\alpha_6\alpha_8(\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8\alpha_{10})^n x_{-1},\\
x_{12n+10} &= \alpha_1\alpha_3\alpha_5\alpha_7\alpha_9(\alpha_1\alpha_3\alpha_5\alpha_7\alpha_9\alpha_{11})^n x_0,
\end{aligned}$$

$$\begin{aligned}
y_{12n-1} &= (\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},\\
y_{12n} &= (\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,\\
y_{12n+1} &= \beta_0(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},\\
y_{12n+2} &= \beta_1(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,\\
y_{12n+3} &= \beta_0\beta_2(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},\\
y_{12n+4} &= \beta_1\beta_3(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,\\
y_{12n+5} &= \beta_0\beta_2\beta_4(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},\\
y_{12n+6} &= \beta_1\beta_3\beta_5(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,\\
y_{12n+7} &= \beta_0\beta_2\beta_4\beta_6(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},\\
y_{12n+8} &= \beta_1\beta_3\beta_5\beta_7(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,\\
y_{12n+9} &= \beta_0\beta_2\beta_4\beta_6\beta_8(\beta_0\beta_2\beta_4\beta_6\beta_8\beta_{10})^n y_{-1},\\
y_{12n+10} &= \beta_1\beta_3\beta_5\beta_7\beta_9(\beta_1\beta_3\beta_5\beta_7\beta_9\beta_{11})^n y_0,
\end{aligned}$$

$$\begin{aligned}
z_{12n-1} &= (\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},\\
z_{12n} &= (\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,\\
z_{12n+1} &= \gamma_0(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},
\end{aligned}$$

$$
\begin{aligned}
z_{12n+2} &= \gamma_1(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,\\
z_{12n+3} &= \gamma_0\gamma_2(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},\\
z_{12n+4} &= \gamma_1\gamma_3(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,\\
z_{12n+5} &= \gamma_0\gamma_2\gamma_4(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},\\
z_{12n+6} &= \gamma_1\gamma_3\gamma_5(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,\\
z_{12n+7} &= \gamma_0\gamma_2\gamma_4\gamma_6(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},\\
z_{12n+8} &= \gamma_1\gamma_3\gamma_5\gamma_7(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,\\
z_{12n+9} &= \gamma_0\gamma_2\gamma_4\gamma_6\gamma_8(\gamma_1\gamma_3\gamma_5\gamma_7\gamma_9\gamma_{11})^n z_{-1},\\
z_{12n+10} &= \gamma_1\gamma_3\gamma_5\gamma_7\gamma_9(\gamma_0\gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10})^n z_0,
\end{aligned}
$$

where

$$
\begin{aligned}
\alpha_0 &= \frac{a_1 y_{-1}}{a x_{-1} + b y_{-1} + c z_{-1}},\\
\alpha_1 &= \frac{a_1 y_0}{a x_0 + b y_0 + c z_0},\\
\alpha_2 &= \frac{a_1 a_2 y_{-1} z_{-1}}{a a_1 x_{-1} y_{-1} + b a_2 y_{-1} z_{-1} + c a_3 x_{-1} z_{-1}},\\
\alpha_3 &= \frac{a_1 a_2 y_0 z_0}{a a_1 x_0 y_0 + b a_2 y_0 z_0 + c a_3 x_0 z_0},\\
\alpha_4 &= \frac{a_1 a_2^2 a_3 z_{-1}}{a a_1^2 a_2 y_{-1} + b a_2^2 a_3 z_{-1} + c a_1 a_3^2 x_{-1}},\\
\alpha_5 &= \frac{a_1 a_2^2 a_3 z_0}{a a_1^2 a_2 y_0 + b a_2^2 a_3 z_0 + c a_1 a_3^2 x_0},\\
\alpha_6 &= \frac{a_1 a_2^2 a_3^2 x_{-1} z_{-1}}{a a_1^2 a_2^2 y_{-1} z_{-1} + b a_2^2 a_3^2 x_{-1} z_{-1} + c a_1^2 a_3^2 x_{-1} y_{-1}},\\
\alpha_7 &= \frac{a_1 a_2^2 a_3^2 x_0 z_0}{a a_1^2 a_2^2 y_0 z_0 + b a_2^2 a_3^2 x_0 z_0 + c a_1^2 a_3^2 x_0 y_0},\\
\alpha_8 &= \frac{a_1 a_2 a_3^2 x_{-1}}{a a_1 a_2^2 z_{-1} + b a_2 a_3^2 x_{-1} + c a_1^2 a_3 y_{-1}},\\
\alpha_9 &= \frac{a_1 a_2 a_3^2 x_0}{a a_1 a_2^2 z_0 + b a_2 a_3^2 x_0 + c a_1^2 a_3 y_0},\\
\alpha_{10} &= \frac{a_1 a_3 x_{-1} y_{-1}}{a a_2 x_{-1} z_{-1} + b a_3 x_{-1} y_{-1} + c a_1 y_{-1} z_{-1}},\\
\alpha_{11} &= \frac{a_1 a_3 x_0 y_0}{a a_2 x_0 z_0 + b a_3 x_0 y_0 + c a_1 y_0 z_0},
\end{aligned}
$$

$$\beta_0 = \frac{a_2 z_{-1}}{a x_{-1} + b y_{-1} + c z_{-1}},$$
$$\beta_1 = \frac{a_2 z_0}{a x_0 + b y_0 + c z_0},$$
$$\beta_2 = \frac{a_2 a_3 x_{-1} z_{-1}}{a a_1 x_{-1} y_{-1} + b a_2 y_{-1} z_{-1} + c a_3 x_{-1} z_{-1}},$$
$$\beta_3 = \frac{a_2 a_3 x_0 z_0}{a a_1 x_0 y_0 + b a_2 y_0 z_0 + c a_3 x_0 z_0},$$
$$\beta_4 = \frac{a_1 a_2 a_3^2 x_{-1}}{a a_1^2 a_2 y_{-1} + b a_2^2 a_3 z_{-1} + c a_1 a_3^2 x_{-1}},$$
$$\beta_5 = \frac{a_1 a_2 a_3^2 x_0}{a a_1^2 a_2 y_0 + b a_2^2 a_3 z_0 + c a_1 a_3^2 x_0},$$
$$\beta_6 = \frac{a_1^2 a_2 a_3^2 x_{-1} y_{-1}}{a a_1^2 a_2^2 y_{-1} z_{-1} + b a_2^2 a_3^2 x_{-1} z_{-1} + c a_1^2 a_3^2 x_{-1} y_{-1}},$$
$$\beta_7 = \frac{a_1^2 a_2 a_3^2 x_0 y_0}{a a_1^2 a_2^2 y_0 z_0 + b a_2^2 a_3^2 x_0 z_0 + c a_1^2 a_3^2 x_0 y_0},$$
$$\beta_8 = \frac{a_1^2 a_2 a_3 y_{-1}}{a a_1 a_2^2 z_{-1} + b a_2 a_3^2 x_{-1} + c a_1^2 a_3 y_{-1}},$$
$$\beta_9 = \frac{a_1^2 a_2 a_3 y_0}{a a_1 a_2^2 z_0 + b a_2 a_3^2 x_0 + c a_1^2 a_3 y_0},$$
$$\beta_{10} = \frac{a_1 a_2 y_{-1} z_{-1}}{a a_2 x_{-1} z_{-1} + b a_3 x_{-1} y_{-1} + c a_1 y_{-1} z_{-1}},$$
$$\beta_{11} = \frac{a_1 a_2 y_0 z_0}{a a_2 x_0 z_0 + b a_3 x_0 y_0 + c a_1 y_0 z_0},$$

$$\gamma_0 = \frac{a_3 x_{-1}}{a x_{-1} + b y_{-1} + c z_{-1}},$$
$$\gamma_1 = \frac{a_3 x_0}{a x_0 + b y_0 + c z_0},$$
$$\gamma_2 = \frac{a_1 a_3 x_{-1} y_{-1}}{a a_1 x_{-1} y_{-1} + b a_2 y_{-1} z_{-1} + c a_3 x_{-1} z_{-1}},$$
$$\gamma_3 = \frac{a_1 a_3 x_0 y_0}{a a_1 x_0 y_0 + b a_2 y_0 z_0 + c a_3 x_0 z_0},$$
$$\gamma_4 = \frac{a_1^2 a_2 a_3 y_{-1}}{a a_1^2 a_2 y_{-1} + b a_2^2 a_3 z_{-1} + c a_1 a_3^2 x_{-1}},$$
$$\gamma_5 = \frac{a_1^2 a_2 a_3 y_0}{a a_1^2 a_2 y_0 + b a_2^2 a_3 z_0 + c a_1 a_3^2 x_0},$$

$$\gamma_6 = \frac{a_1^2 a_2^2 a_3 y_{-1} z_{-1}}{a a_1^2 a_2^2 y_{-1} z_{-1} + b a_2^2 a_3^2 x_{-1} z_{-1} + c a_1^2 a_3^2 x_{-1} y_{-1}},$$
$$\gamma_7 = \frac{a_1^2 a_2^2 a_3 y_0 z_0}{a a_1^2 a_2^2 y_0 z_0 + b a_2^2 a_3^2 x_0 z_0 + c a_1^2 a_3^2 x_0 y_0},$$
$$\gamma_8 = \frac{a_1 a_2^2 a_3 z_{-1}}{a a_1 a_2^2 z_{-1} + b a_2 a_3^2 x_{-1} + c a_1^2 a_3 y_{-1}},$$
$$\gamma_9 = \frac{a_1 a_2^2 a_3 z_0}{a a_1 a_2^2 z_0 + b a_2 a_3^2 x_0 + c a_1^2 a_3 y_0},$$
$$\gamma_{10} = \frac{a_2 a_3 x_{-1} z_{-1}}{a a_2 x_{-1} z_{-1} + b a_3 x_{-1} y_{-1} + c a_1 y_{-1} z_{-1}},$$
$$\gamma_{11} = \frac{a_2 a_3 z_0 x_0}{a a_2 x_0 z_0 + b a_3 x_0 y_0 + c a_1 y_0 z_0}.$$

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