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ANALYTICAL SOLUTIONS TO THE DOUBLE-CHAIN DNA SYSTEM BY TWO COMPUTATIONAL TECHNIQUES

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Abstract. In this article, two algebraic techniques are employed to analyze the double-chain model of deoxyribonucleic acid, a crucial component in the realm of biology. The solutions obtained by these methods include the trigonometric function solution, hyperbolic function solution and rational solution. These methodologies demonstrate significant effectiveness in obtaining exact solutions for many nonlinear differential equations.

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1. INTRODUCTION

Deoxyribonucleic Acid (DNA), is a genetic material that contains all the evolutionary and functional information of living organisms that they need for their reproduction and life. Friedrich Miescher first discovered DNA in the 1800s. But it took a long time for scientists to discover its structure and realize its importance in biology. Structurally, DNA consists of two strands that are parallel and anti-parallel to each other, each filled with numerous nucleotides. These two strands are held together by hydrogen bonds between the bases of each nucleotide pair, keeping them side by side. The physical structure of the two DNA strands as a double helical ladder was first discovered using X-rays in 1953 by James Watson and Francis Crick and was accepted and expanded by scientific communities. This structure allows it to carry biological information. During the last several decades, the structure of DNA has been extensively studied by many scientists. DNA has a complex structure and has many longitudinal, transverse and torsional motions. For this reason, a mathematical model with all its characteristics has not yet been presented. However, to study the structure of DNA, we must provide suitable nonlinear mathematical models. Some scientists have attempted to present some models to describe it, such as the property of the open state in long polynucleotide double helices and possibility of soliton excitations [5], soliton excitations in DNA double helices [22], a coupled base-rotator model for structure and dynamics of DNA [7], two-dimensional discrete model: denaturation and longitudinal wave propagation for DNA dynamics [14], nonlinear dynamics in a new double-chain model of DNA [3], solitary wave solutions for longitudinal and transverse movements of DNA [2, 17], explicit solutions of double-chain DNA dynamical system [12], simulation of the coupled DNA nonlinear dynamical equation bell-shaped [16] and so on.

In this paper, we investigate the double-chain DNA system [18]:

$$\begin{aligned}\Omega_{tt} - c_1^2\Omega_{xx} &= \lambda_1\Omega + \gamma_1\Omega\Gamma + \mu_1\Omega^3 + \beta_1\Omega\Gamma^2,\\ \Gamma_{tt} - c_2^2\Gamma_{xx} &= \lambda_2\Gamma + \gamma_2\Omega^2 + \mu_2\Omega^2\Gamma + \beta_2\Gamma^3 + c_0,\end{aligned}$$

where

$$\begin{aligned}&c_1 = \pm\frac{\chi_1}{\rho},\ c_2 = \pm\frac{\chi_2}{\rho},\ \lambda_1 = \frac{-2\mu}{\rho\sigma h}(h - l_0),\ \lambda_2 = \frac{-2\mu}{\rho\sigma},\ \gamma_1 = 2\gamma_2 = \frac{2\sqrt{2}\mu l_0}{\rho\sigma h^2},\\ &\mu_1 = \mu_2 = \frac{-2\mu l_0}{\rho\sigma h^3},\ \beta_1 = \beta_2 = \frac{4\mu l_0}{\rho\sigma h^3},\ c_0 = \frac{\sqrt{2}\mu}{\rho\sigma}(h - l_0).\end{aligned}$$

Here Ω and Γ indicate the difference in the longitudinal and transverse displacements of the top and bottom strands. This model with two long strands, homogeneous and elastic, demonstrates two polynucleotide chains of the DNA molecule. where χ_1 and χ_2 are the Young's modulus and the tension density of each strand, ρ and σ denote the mass density and the area of transverse cross-section. Also, μ, l_0 and h are the stiffness of the elastic membrane, the height of the membrane in the equilibrium positive and the distance between the two strands, respectively. Exact solutions of nonlinear partial differential equations (NLPDEs) play a critical role in better realizing qualitative features and physical interpretations of many occurrences. Many complicated events can be described by these solutions. For this purpose some techniques have been suggested, such as Kudryashov method [11], $\tan(\phi(\xi)/2)$-expansion method [10], $\exp(-\varphi(\xi))$-expansion method [19], first integral method [8], sine-Gordon method [21], Legendre wavelets [6], $\frac{G'}{G^2}$-expansion method [9], and so on. Many scientists have been applied some methods to study the double-chain DNA system. For examples, Riccati parameterized factorization method [2], $\frac{G'}{G}$-expansion method [12], φ^6-model expansion approach [18], improved generalized Riccati equation mapping method [13], Lie transformation method [20], $\exp(-\varphi(\xi))$-expansion method [1], φ^4-expansion method [4] and so on. Our purpose in this article is to obtain exact solutions to the double-chain DNA system using two algebraic methods.

The remaining parts of this article are constructed as follows. In section 2, we describe the first algebraic method and its application to the double-chain DNA system.

The application of $\frac{G'}{G^2}$-expansion method for the double-chain DNA system is presented in section 2.2. Graphical representations of some solutions is shown in section 4. In the last section, the conclusion is given.

2. THE FIRST ALGEBRAIC METHOD

2.1. *Description of the first method to look exact solutions of NLPDEs*

In this section, we consider the following NLPDE

$$R(\phi,\phi_x,\phi_t,\phi_{xx},\phi_{xt},\phi_{tt},\dots)=0, \tag{2.1}$$

where $\phi=\phi(x,t)$ is an unknown function in two variables x and t. Substituting the travelling wave transformation

$$\xi=\kappa x+\omega t, \tag{2.2}$$

into (2.1), it can be reduced to the following ODE

$$\tilde{R}(\Phi,\Phi',\Phi'',\Phi''',\dots)=0. \tag{2.3}$$

Here $\Phi^{(n)}=\frac{d^n\Phi}{d\xi^n}$. The solution of Eq. (2.3) can be written:

$$\Phi(\xi)=\frac{\sum_{j=0}^{\eta_1}A_j\Theta(\xi)}{\sum_{j=0}^{\eta_2}B_j\Theta(\xi)}, \tag{2.4}$$

where the positive constants η_1 and η_2 can be calculated by considering the homogeneous balance between the highest order derivatives and the highest nonlinear terms of $\Phi(\xi)$ in equation (2.3), and $A_j\,(0\le j\le\eta_1)$, $B_j\,(0\le j\le\eta_2)$ are constants to be found later and $A_{\eta_1},B_{\eta_2}\neq 0$. Here $\Theta=\Theta(\xi)$ satisfies the following ODE

$$\Theta'(\xi)=p+\Theta^2(\xi) \tag{2.5}$$

where p is a constant and which has the following special solutions [15].

Case 1: When $p<0$,

$$\Theta_1(\xi)=-\sqrt{-p}\tanh(\sqrt{-p}\,\xi), \tag{2.6}$$
$$\Theta_2(\xi)=-\sqrt{-p}\coth(\sqrt{-p}\,\xi), \tag{2.7}$$
$$\Theta_3(\xi)=-\sqrt{-p}\tanh(2\sqrt{-p}\,\xi)\pm i\sqrt{-p}\,sech(2\sqrt{-p}\,\xi), \tag{2.8}$$
$$\Theta_4(\xi)=-\sqrt{-p}\coth(2\sqrt{-p}\,\xi)\pm\sqrt{-p}\,csch(2\sqrt{-p}\,\xi), \tag{2.9}$$
$$\Theta_5(\xi)=-\frac{1}{2}\left(\sqrt{-p}\tanh(\frac{\sqrt{-p}}{2}\xi)+\sqrt{-p}\coth(\frac{\sqrt{-p}}{2}\xi)\right). \tag{2.10}$$

Case 2: When $p>0$,

$$\Theta_6(\xi)=\sqrt{p}\tan(\sqrt{p}\,\xi), \tag{2.11}$$
$$\Theta_7(\xi)=-\sqrt{p}\cot(\sqrt{p}\,\xi), \tag{2.12}$$
$$\Theta_8(\xi)=-\sqrt{p}\tan(2\sqrt{p}\,\xi)\pm\sqrt{p}\sec(2\sqrt{p}\,\xi), \tag{2.13}$$

$$\Theta_9(\xi) = -\sqrt{p}\cot(2\sqrt{p}\xi) \pm \sqrt{p}\csc(2\sqrt{p}\xi), \tag{2.14}$$

$$\Theta_{10}(xi) = \frac{1}{2}\left(\sqrt{p}\tan(\frac{\sqrt{p}}{2}\xi) - \sqrt{p}\cot(\frac{\sqrt{p}}{2}\xi)\right). \tag{2.15}$$

Case 3: When $p = 0$,

$$\Theta_{11}(\xi) = -\frac{1}{\xi + d}. \tag{2.16}$$

Where d is a constant.

The exact solutions for NLPDEs can be obtained by following these steps:

First, We substitute in (2.3) the (2.4) with Eq. (2.5). After making this substitution, we obtain a polynomial that involves $\Theta(\xi)$. We then collect the terms with the same powers of $\Theta(\xi)$ and set all the coefficients of the resulting polynomial to zero. This operation yields a set of algebraic equations in terms of $A_j\,(j = 0,1,2,...,\eta_1), B_j\,(j = 0,1,2,...,\eta_2)$, κ and ω. Solving this system, it gives solutions of equation(2.3).

2.2. *Application*

Consider the double-chain DNA system

$$\Omega_{tt} - c_1^2\Omega_{xx} = \lambda_1\Omega + \gamma_1\Omega\Gamma + \mu_1\Omega^3 + \beta_1\Omega\Gamma^2, \tag{2.17}$$

$$\Gamma_{tt} - c_2^2\Gamma_{xx} = \lambda_2\Gamma + \gamma_2\Omega^2 + \mu_2\Omega^2\Gamma + \beta_2\Gamma^3 + c_0. \tag{2.18}$$

By using the transformation

$$\Gamma = a\Omega + b, \tag{2.19}$$

where a and b are constants, Eqs. (2.17)-(2.18) can be converted into the following form

$$\Omega_{tt} - c_1^2\Omega_{xx} = \Omega^3(\mu_1 + \beta_1 a^2) + \Omega^2(2\beta_1 ab + \gamma_1 a) + \Omega\left(\lambda_1 + b\gamma_1 + \beta_1 b^2\right), \tag{2.20}$$

$$\Omega_{tt} - c_2^2\Omega_{xx} = \Omega^3(\mu_2 + \beta_2 a^2) + \Omega^2\left(3\beta_2 ab + \frac{\gamma_2}{a} + \frac{\mu_2 b}{a}\right) + \Omega\left(\lambda_2 + 3\beta_2 b^2\right) + \frac{\lambda_2 b}{a} + \frac{\beta_2 b^3}{a} + \frac{c_0}{a}, \tag{2.21}$$

Eqs. (2.20) and (2.21) are similar for

$$b = \frac{h}{\sqrt{2}}, \chi_1 = \chi_2. \tag{2.22}$$

Now Eqs. (2.20) and (2.21) can be reduced to a single equation as

$$\Omega_{tt} - c_1^2\Omega_{xx} = K\Omega^3 + L\Omega^2 + M\Omega \tag{2.23}$$

where

$$K = \frac{m(4a^2-2)}{h^3},\ L = \frac{6\sqrt{2}am}{h^2},\ M = \left(-\frac{2m}{l_0} + \frac{6m}{h}\right),\ m = \frac{\mu l_0}{\rho\sigma},\ c_1^2 = \frac{\chi_1}{\rho}. \tag{2.24}$$

For obtaining exact solutions of (2.23), We take the travelling wave transformation

$$\Omega(x,t) = \Psi(\xi), \quad \xi = \kappa x + \omega t, \tag{2.25}$$

where κ and ω are constants that should be determined later. Substituting (2.25) into Eq. (2.23), we have

$$(\omega^2 - \kappa^2 c_1^2)\Psi'' - K\Psi^3 - L\Psi^2 - M\Psi = 0, \quad \omega^2 - \kappa^2 c_1^2 \neq 0. \tag{2.26}$$

By balancing Ψ'' with Ψ^3 in (2.26) along with (2.4), we get the below:

$$\eta_1 - \eta_2 + 2 = 3(\eta_1 - \eta_2) \Longrightarrow \eta_1 = \eta_2 + 1. \tag{2.27}$$

Therefore, the exact solution of Eq. (2.26) can be written in the following forms.

Type 1: $\eta_1 = 1$ and $\eta_2 = 0$,

$$\Psi(\xi) = \frac{A_0 + A_1\Theta(\xi)}{B_0}, \tag{2.28}$$

where Θ is the solution of equation (2.5). Substituting Eq. (2.28) along Eq. (2.5) into Eq. (2.26) and equating all of the same powers $\Theta(\xi)$ to zero, we obtain a system of algebraic equations for A_0, A_1, B_0, p, κ and ω. Solving obtained system using *Mathematica*, we obtain

$$A_0 = A_0,\ A_1 = -\frac{1}{\sqrt{-p}}A_0,\ B_0 = -\frac{3K}{L}A_0,$$
$$M = -4p(\omega^2 - \kappa^2 c_1^2),\ K = \frac{2L^2}{9M},\ p < 0. \tag{2.29}$$

By using (2.28), (2.29) and cases (2.6)-(2.10) respectively, we get

$$\Psi_1(x,t) = -\frac{(1 + \tanh(\sqrt{-p}(\kappa x + \omega t)))L}{3K},$$
$$\Psi_2(x,t) = -\frac{(1 + \coth(\sqrt{-p}(\kappa x + \omega t)))L}{3K},$$
$$\Psi_3(x,t) = -\frac{[1 + \tanh(2\sqrt{-p}(\kappa x + \omega t)) \mp isech(2\sqrt{-p}(\kappa x + \omega t))]L}{3K},$$
$$\Psi_4(x,t) = -\frac{[1 + \coth(2\sqrt{-p}(\kappa x + \omega t)) \mp csch(2\sqrt{-p}(\kappa x + \omega t))]L}{3K},$$
$$\Psi_5(x,t) = \frac{[1 + \frac{1}{2}(\tanh(\frac{\sqrt{-p}}{2}(\kappa x + \omega t)) + \coth(\frac{\sqrt{-p}}{2}(\kappa x + \omega t)))]L}{3K}.$$

Type 2: $\eta_1 = 2$ and $\eta_2 = 1$,

$$\Psi(\xi) = \frac{A_0 + A_1\Theta(\xi) + A_2\Theta^2(\xi)}{B_0 + B_1\Theta(\xi)}, \tag{2.30}$$

Substituting Eq. (2.30) along Eq. (2.5) into Eq. (2.26), same as before, we obtain

- *Set* 1 : $A_0 = A_0,\ A_1 = \frac{2\sqrt{5}}{3\sqrt{-2p}}A_0,\ A_2 = -\frac{1}{3p}A_0,\ B_0 = -\frac{12K}{5L}A_0,$

$$B_1=-\frac{6\sqrt{5}K}{5\sqrt{-2p}}A_0,\ M=-16p(\omega^2-\kappa^2c_1^2),\ K=\frac{2L^2}{9M},\ p<0. \tag{2.31}$$

By using (2.30), (2.31) and cases (2.6)-(2.10) respectively, we get

$$\Psi_1(x,t)=\frac{1-[\frac{\sqrt{10}}{3}-\frac{1}{3}\tanh(\sqrt{-p}(\kappa x+\omega t))]\tanh(\sqrt{-p}(\kappa x+\omega t))}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}\tanh(\sqrt{-p}(\kappa x+\omega t))},$$

$$\Psi_2(x,t)=\frac{1-[\frac{\sqrt{10}}{3}-\frac{1}{3}\coth(\sqrt{-p}(\kappa x+\omega t))]\coth(\sqrt{-p}(\kappa x+\omega t))}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}\coth(\sqrt{-p}(\kappa x+\omega t))},$$

$$\begin{aligned}\Psi_3(x,t)=&\left(\frac{[-\frac{\sqrt{10}}{3}+\frac{1}{3}(-\tanh(2\sqrt{-p}(\kappa x+\omega t))\pm i\,sech(2\sqrt{-p}(\kappa x+\omega t)))]}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}(-\tanh(2\sqrt{-p}(\kappa x+\omega t))\pm i\,sech(2\sqrt{-p}(\kappa x+\omega t)))}\right)\\&\times(-\tanh(2\sqrt{-p}(\kappa x+\omega t))\pm i\,sech(2\sqrt{-p}(\kappa x+\omega t)))\\&+\frac{1}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}(-\tanh(2\sqrt{-p}(\kappa x+\omega t))\pm i\,sech(2\sqrt{-p}(\kappa x+\omega t)))},\end{aligned}$$

$$\begin{aligned}\Psi_4(x,t)=&\left(\frac{[-\frac{\sqrt{10}}{3}+\frac{1}{3}(-\coth(2\sqrt{-p}(\kappa x+\omega t))\pm csch(2\sqrt{-p}(\kappa x+\omega t)))]}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}(-\coth(2\sqrt{-p}(\kappa x+\omega t))\pm csch(2\sqrt{-p}(\kappa x+\omega t)))}\right)\\&\times(-\coth(2\sqrt{-p}(\kappa x+\omega t))\pm csch(2\sqrt{-p}(\kappa x+\omega t)))\\&+\frac{1}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}(-\coth(2\sqrt{-p}(\kappa x+\omega t))\pm csch(2\sqrt{-p}(\kappa x+\omega t)))},\end{aligned}$$

$$\begin{aligned}\Psi_5(x,t)=&\left(\frac{[\frac{\sqrt{10}}{3}+\frac{1}{6}(\tanh(\frac{\sqrt{-p}}{2}(\kappa x+\omega t))+\coth(\frac{\sqrt{-p}}{2}(\kappa x+\omega t)))]}{\frac{24K}{5L}+\frac{12}{\sqrt{10}}(\tanh(\frac{\sqrt{-p}}{2}(\kappa x+\omega t))+\coth(\frac{\sqrt{-p}}{2}(\kappa x+\omega t)))}\right)\\&\times(\tanh(\frac{\sqrt{-p}}{2}(\kappa x+\omega t))+\coth(\frac{\sqrt{-p}}{2}(\kappa x+\omega t)))\\&+\frac{1}{\frac{12K}{5L}+\frac{6}{\sqrt{10}}(\tanh(\frac{\sqrt{-p}}{2}(\kappa x+\omega t))+\coth(\frac{\sqrt{-p}}{2}(\kappa x+\omega t)))}.\end{aligned}$$

- *Set* 2 : $A_0=A_0,\ A_1=0,\ A_2=\frac{1}{p}A_0,\ B_0=-\frac{2L}{3M}A_0,\ B_1=\frac{\sqrt{-6KL}}{\sqrt{pM}}A_0,$

$$M=-4p(\omega^2-\kappa^2c_1^2),\ K=K,\ p>0. \tag{2.32}$$

By using (2.30), (2.32) and cases (2.11)-(2.15) respectively, we get

$$\Psi_6(x,t)=\frac{1+\tan^2(\sqrt{p}(\kappa x+\omega t))}{-\frac{2L}{3M}+\frac{\sqrt{-6KL}}{\sqrt{M}}\tan(\sqrt{p}(\kappa x+\omega t))},$$

$$\Psi_7(x,t)=\frac{1+\cot^2(\sqrt{p}(\kappa x+\omega t))}{-\frac{2L}{3M}-\frac{\sqrt{-6KL}}{\sqrt{M}}\cot(\sqrt{p}(\kappa x+\omega t))},$$

$$\Psi_8(x,t)=\frac{1+(-\tan(2\sqrt{p}(\kappa x+\omega t))\pm\sec(2\sqrt{p}(\kappa x+\omega t)))^2}{-\frac{2L}{3M}+\frac{\sqrt{-6KL}}{\sqrt{M}}(-\tan(2\sqrt{p}(\kappa x+\omega t))\pm\sec(2\sqrt{p}(\kappa x+\omega t)))},$$

$$\Psi_9(x,t)=\frac{1+(-\cot(2\sqrt{p}(\kappa x+\omega t))\pm\csc(2\sqrt{p}(\kappa x+\omega t)))^2}{-\frac{2L}{3M}+\frac{\sqrt{-6KL}}{\sqrt{M}}(-\cot(2\sqrt{p}(\kappa x+\omega t))\pm\csc(2\sqrt{p}(\kappa x+\omega t)))},$$

$$\Psi_{10}(x,t)=\frac{1+\frac{1}{4}(\tan(\frac{\sqrt{p}}{2}(\kappa x+\omega t))-\cot(\frac{\sqrt{p}}{2}(\kappa x+\omega t)))^2}{-\frac{2L}{3M}+\frac{\sqrt{-6KL}}{2\sqrt{M}}(\tan(\frac{\sqrt{p}}{2}(\kappa x+\omega t))-\cot(\frac{\sqrt{p}}{2}(\kappa x+\omega t)))}.$$

$$\bullet\, Set\, 3: \quad A_0=A_0,\ A_1=0,\ A_2=\frac{1}{p}A_0,\ B_0=-\frac{3K}{L}A_0,\ B_1=\frac{\sqrt{3K}}{\sqrt{-pL}}A_0,$$
$$M=-4p(\omega^2-\kappa^2c_1^2),\ K=\frac{2L^2}{9M},\ p<0. \tag{2.33}$$

By using (2.30), (2.33) and cases (2.6)-(2.10) respectively, we get

$$\Psi_{11}(x,t)=\frac{1-\tanh^2(\sqrt{-p}(\kappa x+\omega t))}{-\frac{3K}{L}-\frac{3K}{\sqrt{L}}\tanh(\sqrt{-p}(\kappa x+\omega t))},$$

$$\Psi_{12}(x,t)=\frac{1-\coth^2(\sqrt{-p}(\kappa x+\omega t))}{-\frac{3K}{L}-\frac{3K}{\sqrt{L}}\coth(\sqrt{-p}(\kappa x+\omega t))},$$

$$\Psi_{13}(x,t)=\frac{1-(-\tanh(2\sqrt{-p}(\kappa x+\omega t))\pm i\,sech(2\sqrt{-p}(\kappa x+\omega t)))^2}{-\frac{3K}{L}-\frac{3K}{\sqrt{L}}(-\tanh(2\sqrt{-p}(\kappa x+\omega t))\pm i\,sech(2\sqrt{-p}(\kappa x+\omega t)))},$$

$$\Psi_{14}(x,t)=\frac{1-(-\coth(2\sqrt{-p}(\kappa x+\omega t))\pm csch(2\sqrt{-p}(\kappa x+\omega t)))^2}{-\frac{3K}{L}-\frac{3K}{\sqrt{L}}(-\coth(2\sqrt{-p}(\kappa x+\omega t))\pm csch(2\sqrt{-p}(\kappa x+\omega t)))},$$

$$\Psi_{15}(x,t)=\frac{1-\frac{1}{4}(\tanh(\frac{\sqrt{-p}}{2}(\kappa x+\omega t))+\coth(\frac{\sqrt{-p}}{2}(\kappa x+\omega t)))^2}{-\frac{3K}{L}-\frac{3K}{2\sqrt{L}}(\tanh(\frac{\sqrt{-p}}{2}(\kappa x+\omega t))+\coth(\frac{\sqrt{-p}}{2}(\kappa x+\omega t)))}.$$

$$\bullet\, Set\, 4: \quad A_0=A_0,\ A_1=\frac{3\sqrt{2(\omega^2-\kappa^2c_1^2)}}{\sqrt{M}}A_0,\ A_2=\frac{6(\omega^2-\kappa^2c_1^2)}{M}A_0,$$
$$B_0=-\frac{2K}{L}A_0,\ B_1=-\frac{6K\sqrt{2(\omega^2-\kappa^2c_1^2)}}{L\sqrt{M}}A_0,\ K=\frac{L^2}{4M},\ p=0. \tag{2.34}$$

By using (2.30), (2.34) and case (2.16), we get

$$\Psi_{16}(x,t)=\frac{1+\left(3\sqrt{2}+\frac{6\sqrt{(\omega^2-\kappa^2c_1^2)}}{\sqrt{M}(\kappa x+\omega t+d)}\right)\frac{\sqrt{(\omega^2-\kappa^2c_1^2)}}{\sqrt{M}(\kappa x+\omega t+d)}}{-\frac{2K}{L}-\frac{6\sqrt{2}K}{L}\frac{\sqrt{(\omega^2-\kappa^2c_1^2)}}{\sqrt{M}(\kappa x+\omega t+d)}}.$$

3. THE SECOND ALGEBRAIC METHOD

3.1. *Description of extended $\frac{G'}{G^2}$-expansion method*

In this section, we employ the $\frac{G'}{G^2}$-expansion method to obtain the exact solutions of NLPDEs.

For the following NLPDE

$$R(\phi,\phi_x,\phi_t,\phi_{xx},\phi_{xt},\phi_{tt},\ldots)=0, \tag{3.1}$$

where $\phi=\phi(x,t)$ is an unknown function in two variables x and t. Substituting the following transformation

$$\xi=\kappa x+\omega t, \tag{3.2}$$

into (3.1), it can be transformed to the following ODE

$$\tilde{R}(\Phi,\Phi',\Phi'',\Phi''',\ldots)=0. \tag{3.3}$$

Here $\Phi^{(n)}=\frac{d^n\Phi}{d\xi^n}$. We assume the exact solution of Eq. (3.3) as follow

$$\Phi(\xi)=\sum_{i=-m}^{m}A_i\left(\frac{G'(\xi)}{G^2(\xi)}\right)^i, \tag{3.4}$$

where $A_i\,(A_m\neq 0)$ are constants and $G(\xi)$ satisfies the following ODE:

$$G''(\xi)\,G^2(\xi)-2G(\xi)\,G'^2(\xi)=p\,G^4(\xi)+q\,G'(\xi)\,G^2(\xi)+r\,G'^2(\xi). \tag{3.5}$$

We know the Eq. (3.5) has the following special solutions:

Case 1. If $pr>0$ and $q=0$,

$$\frac{G'}{G^2}=\frac{\sqrt{pr}}{p}\left(\frac{C_1\cos\sqrt{pr}\xi+C_2\sin\sqrt{pr}\xi}{C_1\sin\sqrt{pr}\xi-C_2\cos\sqrt{pr}\xi}\right). \tag{3.6}$$

Case 2. If $pr<0$ and $q=0$,

$$\frac{G'}{G^2}=-\frac{\sqrt{-pr}}{p}\left(\frac{C_1\sinh 2\sqrt{-pr}\xi+C_2\cosh 2\sqrt{-pr}\xi+C_2}{C_1\cosh 2\sqrt{-pr}\xi+C_2\sinh 2\sqrt{-pr}\xi-C_2}\right). \tag{3.7}$$

Case 3. If $p=q=0$ and $r\neq 0$,

$$\frac{G'}{G^2}=-\frac{C_1}{r(C_1\xi+C_2)}. \tag{3.8}$$

Case 4. If $q \neq 0$ and $q^2 - 4pr \geq 0$,

$$\frac{G'}{G^2} = -\frac{q}{2r} - \left(\frac{\sqrt{q^2-4pr}(C_1 \sinh(\frac{\sqrt{q^2-4pr}}{2})\xi + C_2 \cosh(\frac{\sqrt{q^2-4pr}}{2})\xi)}{2r(C_1 \cosh(\frac{\sqrt{q^2-4pr}}{2})\xi + C_2 \sinh(\frac{\sqrt{q^2-4pr}}{2})\xi)} \right). \quad (3.9)$$

Case 5. If $q \neq 0$ and $q^2 - 4pr < 0$,

$$\frac{G'}{G^2} = -\frac{q}{2r} - \left(\frac{\sqrt{4pr-q^2}(-C_1 \sinh(\frac{\sqrt{4pr-q^2}}{2})\xi + C_2 \cosh(\frac{\sqrt{4pr-q^2}}{2})\xi)}{2r(C_1 \cosh(\frac{\sqrt{4pr-q^2}}{2})\xi + C_2 \sinh(\frac{\sqrt{4pr-q^2}}{2})\xi)} \right). \quad (3.10)$$

Where C_1 and C_2 are arbitrary constants. The following steps can be used to obtain the exact solutions of NLPDEs.

First, substitute (3.4) into (3.3) using (3.5). Next, collect the terms with the same powers of $\frac{G'}{G^2}$ in the resulting polynomial. Then, set all the coefficients of this polynomial to zero to obtain a system of algebraic equations in terms of $A_j(-m \leq j \leq m)$, p, q and r. Finally, solve this system to obtain the solutions of (3.3).

3.2. *Application*

For the double-chain DNA system

$$\Omega_{tt} - c_1^2 \Omega_{xx} = \lambda_1 \Omega + \gamma_1 \Omega \Gamma + \mu_1 \Omega^3 + \beta_1 \Omega \Gamma^2, \quad (3.11)$$

$$\Gamma_{tt} - c_2^2 \Gamma_{xx} = \lambda_2 \Gamma + \gamma_2 \Omega^2 + \mu_2 \Omega^2 \Gamma + \beta_2 \Gamma^3 + c_0, \quad (3.12)$$

similar to the first method, we can obtain

$$(\omega^2 - \kappa^2 c_1^2)\Psi'' - K\Psi^3 - L\Psi^2 - M\Psi = 0, \quad \omega^2 - \kappa^2 c_1^2 \neq 0. \quad (3.13)$$

Balancing Ψ'' with Ψ^3 in (3.13) gives n=1. Therefore, the exact solution of Eq. (3.13) can be written in the form:

$$\Psi(\xi) = A_0 + A_1 \left(\frac{G'}{G^2}\right) + A_{-1} \left(\frac{G'}{G^2}\right)^{-1}, \quad A_1 \neq 0. \quad (3.14)$$

Therefore, we have

$$\bullet \textit{Set 1}: \quad A_0 = \frac{L}{3K}\left(\frac{q}{\sqrt{q^2-4pr}} - 1\right), A_1 = \frac{2Lr}{3Kq}\sqrt{\frac{4pr}{q^2-4pr}+1}, A_{-1} = 0,$$
$$M = (q^2-4pr)(\omega^2 - \kappa^2 c_1^2), K = \frac{2L^2}{9M}, q^2 - 4pr \neq o, q \neq o, r \neq o. \quad (3.15)$$

By using (3.15), (3.14) and case (3.9), we get the general hyperbolic function solutions

$$\Psi_1(x,t) = \frac{L}{3K}\left(\frac{q}{\sqrt{q^2-4pr}} - 1\right) + \frac{L}{3Kq}\sqrt{\frac{4pr}{q^2-4pr}+1}\Bigg[-q$$

$$-\left(\frac{\sqrt{q^2-4pr}\left[C_1\sinh(\frac{\sqrt{q^2-4pr}}{2}(\kappa x+\omega t))+C_2\cosh(\frac{\sqrt{q^2-4pr}}{2}(\kappa x+\omega t))\right]}{C_1\cosh(\frac{\sqrt{q^2-4pr}}{2}(\kappa x+\omega t))+C_2\sinh(\frac{\sqrt{q^2-4pr}}{2}(\kappa x+\omega t))}\right)\Bigg],$$

In particular, if we choose $C_2=0$, then this solution gives the solitary wave solution

$$\Psi_2(x,t)=\frac{L}{3K}\left(\frac{q}{\sqrt{q^2-4pr}}-1\right)+\frac{L}{3Kq}\sqrt{\frac{4pr}{q^2-4pr}+1}$$
$$\times\left[-q-\sqrt{q^2-4pr}\tanh\left(\frac{\sqrt{q^2-4pr}}{2}(\kappa x+\omega t)\right)\right].$$

By using (3.15), (3.14) and case (3.10), we get the general hyperbolic function solutions

$$\Psi_3(x,t)=\frac{L}{3K}\left(\frac{q}{\sqrt{4pr-q^2}}-1\right)+\frac{L}{3Kq}\sqrt{\frac{4pr}{4pr-q^2}+1}\Bigg[-q+$$
$$\left(\frac{\sqrt{4pr-q^2}\left[C_1\sinh\left(\frac{\sqrt{4pr-q^2}}{2}(\kappa x+\omega t)\right)-C_2\cosh\left(\frac{\sqrt{4pr-q^2}}{2}(\kappa x+\omega t)\right)\right]}{C_1\cosh(\frac{\sqrt{4pr-q^2}}{2}(\kappa x+\omega t))+C_2\sinh(\frac{\sqrt{4pr-q^2}}{2}(\kappa x+\omega t))}\right)\Bigg],$$

In particular, if we choose $C_2=0$, then this solution gives the solitary wave solution

$$\Psi_4(x,t)=\frac{L}{3K}\left(\frac{q}{\sqrt{4pr-q^2}}-1\right)+\frac{L}{3Kq}\sqrt{\frac{4pr}{4pr-q^2}+1}$$
$$\times\left[-q+\sqrt{4pr-q^2}\tanh\left(\frac{\sqrt{4pr-q^2}}{2}(\kappa x+\omega t)\right)\right],$$

$$\bullet\, Set\,2:\quad A_0=-\frac{L}{3K},\ A_1=\frac{\sqrt{Mr}}{2\sqrt{Kp}},\ A_{-1}=\frac{\sqrt{Mp}}{2\sqrt{K}},$$
$$M=-4pr(\omega^2-\kappa^2c_1^2),\ K=\frac{2L^2}{9M},\ q=o,\ p\neq o,\ r\neq o. \tag{3.16}$$

By using (3.16), (3.14) and case (3.6), we get the general trigonometric function solutions

$$\Psi_5(x,t)=-\frac{L}{3K}+\frac{\sqrt{Mr}}{2\sqrt{Kp}}\left[\left(\frac{C_1\cos(\sqrt{pr}(\kappa x+\omega t))+C_2\sin(\sqrt{pr}(\kappa x+\omega t))}{C_1\sin(\sqrt{pr}(\kappa x+\omega t))-C_2\cos(\sqrt{pr}(\kappa x+\omega t))}\right)\right]$$
$$+\frac{\sqrt{Mp}}{2\sqrt{Kr}}\left[\left(\frac{C_1\sin(\sqrt{pr}(\kappa x+\omega t))-C_2\cos(\sqrt{pr}(\kappa x+\omega t))}{C_1\cos(\sqrt{pr}(\kappa x+\omega t))+C_2\sin(\sqrt{pr}(\kappa x+\omega t))}\right)\right].$$

In particular, if we choose $C_2 = 0$, then this solution gives the solitary wave solution

$$\Psi_6(x,t) = -\frac{L}{3K} + \frac{\sqrt{Mr}}{2\sqrt{Kp}}\cot(\sqrt{pr}(\kappa x+\omega t)) + \frac{\sqrt{Mp}}{2\sqrt{Kr}}\tan(\sqrt{pr}(\kappa x+\omega t)).$$

By using (3.16), (3.14) and case (3.7), we get the general trigonometric function solutions

$$\begin{aligned}\Psi_7(x,t) = &-\frac{L}{3K}\\ &-\frac{\sqrt{-Mr}}{2\sqrt{Kp}}\left(\frac{C_1\sinh(2\sqrt{-pr}(\kappa x+\omega t))+C_2\cosh(2\sqrt{-pr}(\kappa x+\omega t))+C_2}{C_1\cosh(2\sqrt{-pr}(\kappa x+\omega t))+C_2\sinh(2\sqrt{-pr}(\kappa x+\omega t))-C_2}\right)\\ &+\frac{\sqrt{-Mp}}{2\sqrt{Kr}}\left(\frac{C_1\cosh(2\sqrt{-pr}(\kappa x+\omega t))+C_2\sinh(2\sqrt{-pr}(\kappa x+\omega t))-C_2}{C_1\sinh(2\sqrt{-pr}(\kappa x+\omega t))+C_2\cosh(2\sqrt{-pr}(\kappa x+\omega t))+C_2}\right).\end{aligned}$$

In particular, if we choose $C_2 = 0$, then this solution gives the solitary wave solution

$$\Psi_8(x,t) = -\frac{L}{3K} - \frac{\sqrt{-Mr}}{2\sqrt{Kp}}\tanh(2\sqrt{-pr}(\kappa x+\omega t)) + \frac{\sqrt{-Mp}}{2\sqrt{Kr}}\coth(2\sqrt{-pr}(\kappa x+\omega t)).$$

$$\begin{aligned}\bullet\, Set\,3: \quad & A_0 = \frac{2L}{7K},\ A_1 = \frac{26Lr}{21Kq},\ A_{-1} = \frac{26Lp}{11Kq},\\ & M = -16pr(\omega^2-\kappa^2c_1^2),\ K = \frac{2L^2}{9M},\ q\neq o,\ p\neq o,\ r\neq o.\end{aligned} \tag{3.17}$$

By using (3.17), (3.14) and case (3.9), we get the general hyperbolic function solutions

$$\begin{aligned}\Psi_9(x,t) = &\frac{2L}{7K} - \frac{13L}{21Kq}\left(q+\frac{\sqrt{Z}[C_1\sinh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))+C_2\cosh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))]}{C_1\cosh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))+C_2\sinh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))}\right)\\ &-\frac{26Lp}{11Kq}\left(\frac{q}{2r}+\frac{\sqrt{Z}[C_1\sinh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))+C_2\cosh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))]}{2r(C_1\cosh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))+C_2\sinh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t)))}\right)^{-1},\end{aligned}$$

In particular, if we choose $C_2 = 0$, then this solution gives the solitary wave solution

$$\begin{aligned}\Psi_{10}(x,t) = &\frac{2L}{7K} - \frac{13L}{21Kq}\left(q+\sqrt{Z}\tanh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))\right)\\ &-\frac{26Lp}{11Kq}\left(\frac{q}{2r}+\frac{\sqrt{Z}}{2r}\tanh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))\right)^{-1}.\end{aligned}$$

Where $Z = q^2 - 4pr$.

By using (3.17), (3.14) and case (3.10), we get the general hyperbolic function solutions

$$\Psi_{11}(x,t)=\frac{2L}{7K}-\frac{13L}{21Kq}\left(q-\frac{\sqrt{-Z}[C_1\sinh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))-C_2\cosh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))]}{C_1\cosh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))+C_2\sinh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))}\right)$$
$$-\frac{26Lp}{11Kq}\left(\frac{q}{2r}-\frac{\sqrt{-Z}[C_1\sinh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))-C_2\cosh(\frac{\sqrt{Z}}{2}(\kappa x+\omega t))]}{2r(C_1\cosh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))+C_2\sinh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t)))}\right)^{-1},$$

In particular, if we choose $C_2=0$, then this solution gives the solitary wave solution

$$\Psi_{12}(x,t)=\frac{2L}{7K}-\frac{13L}{21Kq}\left(q-\sqrt{-Z}\tanh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))\right)$$
$$-\frac{26Lp}{11Kq}\left(\frac{q}{2r}-\frac{\sqrt{-Z}}{2r}\tanh(\frac{\sqrt{-Z}}{2}(\kappa x+\omega t))\right)^{-1}.$$

Where $Z=q^2-4pr$.

4. Graphical presentments of some solutions

In figures 1-4 and 5-7, we plot 2D and 3D graphics of some obtained solutions to Eq. (2.23) in the first and second methods respectively, which denote the dynamics of solutions with appropriate parameters selection.

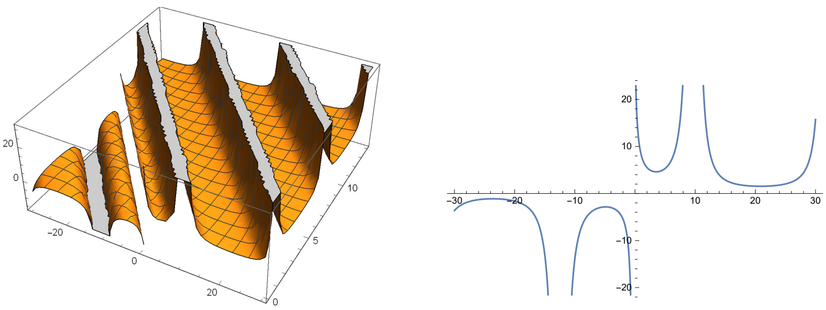

FIGURE 1. The 3D and 2D surfaces of $\Psi_1(x,t)$ (Set 1) for the values $p=-0.5$, $\kappa=0.2$, $\omega=0.7$, $K=-0.05$, $L=-3.9$, and $t=0.4$.

5. Concluding remarks

The purpose of this paper is to study the double-chain DNA. A travelling wave transformation has been utilized on this model to convert it into an ordinary differential equation. In this paper, two algebraic methods were successfully used to study the double-chain DNA system. The exact solutions obtained include the trigonometric function solutions, rational solutions and hyperbolic function solutions. This

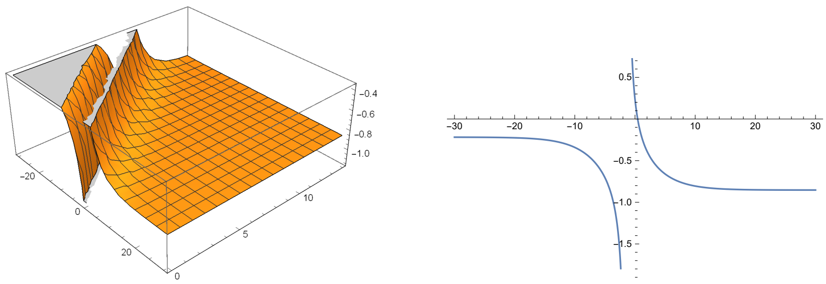

FIGURE 2. The 3D and 2D surfaces of $\Psi_4(x,t)$ (Set 1) for the values $p = -0.5$, $\kappa = 0.2$, $\omega = 0.7$, $K = -0.05$, $L = -3.9$, and $t = 0.4$.

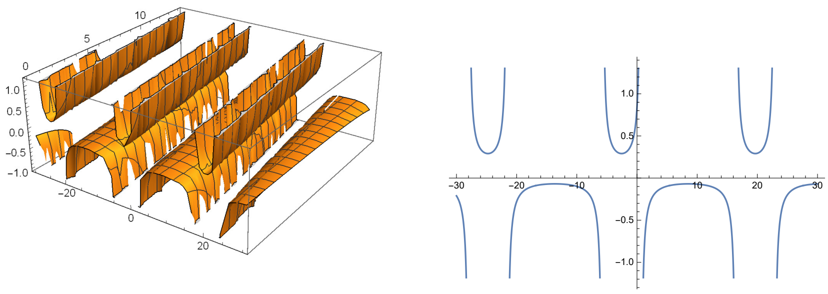

FIGURE 3. The 3D and 2D surfaces of $\Psi_8(x,t)$ (Set 2) for the values $p = 0.5$, $\kappa = 0.2$, $\omega = 0.1$, $K = -2$, $L = 1$, $M = 0.06$ and $t = 0.4$.

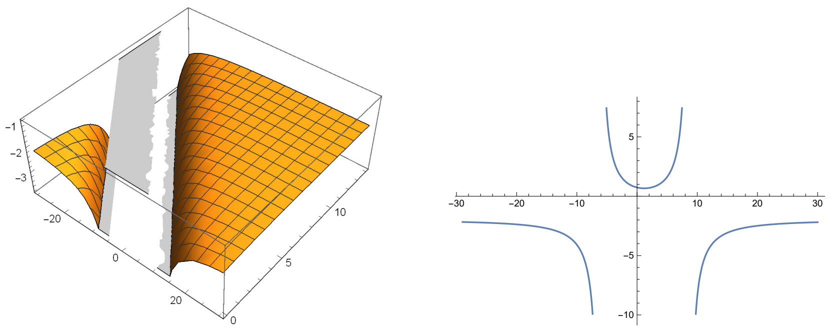

FIGURE 4. The 3D and 2D surfaces of $\Psi_{16}(x,t)$ (Set 4) for the values $p = 0$, $\kappa = 0.2$, $\omega = 0.7$, $K = 0.25$, $L = 1$, $M = 1$, $c_1 = 0.2$, $d = -2$ and $t = 0.4$.

research illustrates the high effectiveness and practical utility of these methods in obtaining exact solutions for various types of nonlinear differential equations. We used Mathematica for computations.

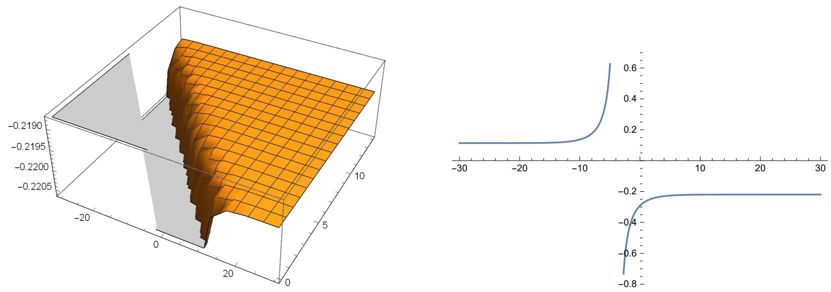

FIGURE 5. The 3D and 2D surfaces of $\Psi_1(x,t)$ (Set 1) for the values $p=-0.5$, $q=3$, $r=-2$, $\kappa=0.2$, $\omega=0.7$, $K=2$, $L=1$, $M=1$, $C_1=1$, $C_2=2$ and $t=0.4$.

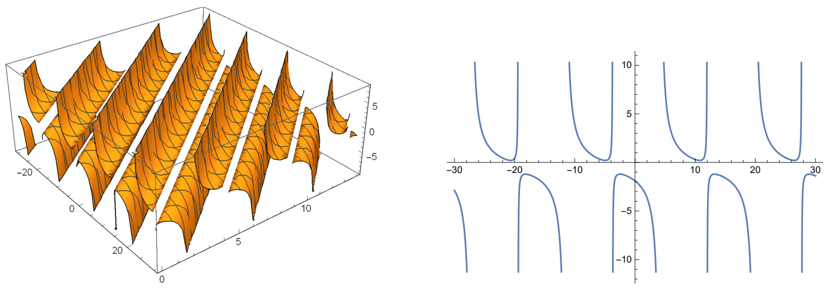

FIGURE 6. The 3D and 2D surfaces of $\Psi_5(x,t)$ (Set 2) for the values $p=0.5$, $q=0$, $r=2$, $\kappa=0.2$, $\omega=0.7$, $K=2$, $L=1$, $M=1$, $C_1=-2$, $C_2=1$ and $t=0.4$.

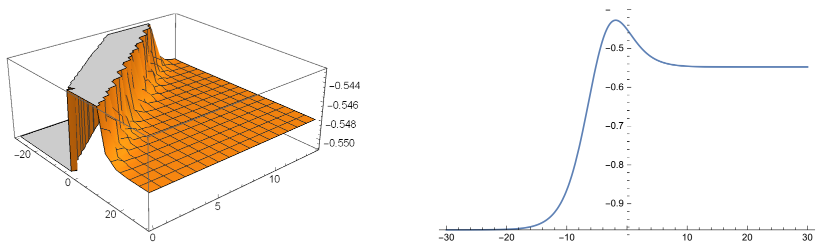

FIGURE 7. The 3D and 2D surfaces of $\Psi_{10}(x,t)$ (Set 3) for the values $p=-0.5$, $q=3$, $r=-2$, $\kappa=0.2$, $\omega=0.7$, $K=2$, $L=1$, $M=1$, $C_1=1$, $C_2=0$ and $t=0.4$.

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