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APPLICATIONS OF HORADAM POLYNOMIALS ON A NEW FAMILY OF BI-PRESTARLIKE FUNCTIONS

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Abstract. In this article, we introduce and investigate a new family of analytic and bi-prestarlike functions by using the Horadam polynomials defined in the open unit disk U. We determine upper bounds for the first two coefficients $|a_2|$ and $|a_3|$ and solve Fekete-Szegő problem of functions that belong to this family. Also, we point out several certain special cases for our results.

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1. INTRODUCTION AND PRELIMINARIES

Indicate by $\mathcal{A}$ the collection of analytic functions in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ that have the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Further, let S stand for the subclass of $\mathcal{A}$ containing of functions in U satisfying (1.1) which are univalent in U.

A function $f \in \mathcal{A}$ is called starlike of order θ $(0 \leq \theta < 1)$, if

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \theta, \quad (z \in U).$$

For $f \in \mathcal{A}$ given by (1.1) and $g \in \mathcal{A}$ defined by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

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the Hadamard product of f and g is defined (as usual) by

$$(f*g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad (z \in U).$$

Ruscheweyh [14] defined and considered the family of prestarlike functions of order θ, which are the functions f such that $f * I_\theta$ is a starlike function of order θ, where

$$I_\theta(z) = \frac{z}{(1-z)^{2(1-\theta)}}, \quad (0 \le \theta < 1, z \in U).$$

The function I_θ can be written in the form:

$$I_\theta(z) = z + \sum_{n=2}^{\infty} \varphi_n(\theta) z^n,$$

where

$$\varphi_n(\theta) = \frac{\prod_{i=2}^{n} (i - 2\theta)}{(n-1)!}, \quad n \ge 2.$$

We note that $\varphi_n(\theta)$ is a decreasing function in θ and satisfies

$$\lim_{n\to\infty} \varphi_n(\theta) = \begin{cases} \infty, & if\, \theta < \frac{1}{2} \\ 1, & if\, \theta = \frac{1}{2} \\ 0, & if\, \theta > \frac{1}{2} \end{cases}.$$

According to the Koebe one-quarter theorem (see [6]) every function $f \in S$ has an inverse f^{-1} which satisfies $f^{-1}(f(z)) = z$, $(z \in U)$ and $f(f^{-1}(w)) = w$, $(|w| < r_0(f), r_0(f) \ge \frac{1}{4})$, where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots. \quad (1.2)$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in U if both f and f^{-1} are univalent in U. Let Σ stands for the class of bi-univalent functions in U given by (1.1). Srivastava et al. [19] revived the study of analytic and bi-univalent functions in recent years, was followed by such works as those by Bulut [4], Adegani and et al. [1], Caglar et al. [5] and others (see, for example [15, 17, 18, 20]). We notice that the class Σ is not empty. For example, the functions z, $\frac{z}{1-z}$, $-\log(1-z)$ and $\frac{1}{2}\log\frac{1+z}{1-z}$ are members of Σ. However, the Koebe function is not a member of Σ. Until now, the coefficient estimate problem for each of the following Taylor-Maclaurin coefficients $|a_n|$, $(n = 3, 4, \cdots)$ for functions $f \in \Sigma$ is still an open problem.

Let the functions f and g be analytic in U. We say that the function f is said to be subordinate to g, if there exists a Schwarz function w analytic in U with $w(0) = 0$ and $|w(z)| < 1$ $(z \in U)$ such that $f(z) = g(w(z))$. This subordination is denoted by $f \prec g$ or $f(z) \prec g(z)$ $(z \in U)$. It is well known (see [13]) that if the function g is univalent in U, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(U) \subset g(U)$.

The Horadam polynomials $h_n(r)$ are defined by the following repetition relation (see [8]):

$$h_n(r) = prh_{n-1}(r) + qh_{n-2}(r) \quad (r \in \mathbb{R}, n \in \mathbb{N} = \{1,2,3,\cdots\}), \tag{1.3}$$

with $h_1(r) = a \quad and \quad h_2(r) = br$, for some real constant a,b,p and q. The characteristic equation of repetition relation (1.3) is $t^2 - prt - q = 0$. This equation has two real roots $x = \frac{pr+\sqrt{p^2r^2+4q}}{2}$ and $y = \frac{pr-\sqrt{p^2r^2+4q}}{2}$.

Remark 1. For particular values of a, b, p and q, the Horadam polynomial $h_n(r)$ reduces to several known polynomials listed below:

(1) the Fibonacci polynomials $F_n(r)$ $(a = b = p = q = 1)$.
(2) the Lucas polynomials $L_n(r)$ $(a = 2$ and $b = p = q = 1)$.
(3) the Pell polynomials $P_n(r)$ $(a = q = 1$ and $b = p = 2)$.
(4) the Pell-Lucas polynomials $Q_n(r)$ $(a = b = p = 2$ and $q = 1)$.
(5) the Chebyshev polynomials $T_n(r)$ of the first kind $(a = b = 1,\ p = 2$ and $q = -1)$.
(6) the Chebyshev polynomials $U_n(r)$ of the second kind $(a = 1,\ b = p = 2$ and $q = -1)$.

These polynomials, the families of orthogonal polynomials and other special polynomials as well as their generalizations are potentially important in a variety of disciplines in many of sciences, specially in mathematics, statistics and physics. For more information associated with these polynomials see [7,8,10,11,21].

The generating function of the Horadam polynomials $h_n(r)$ (see [9]) is given by

$$\Pi(r,z) = \sum_{n=1}^{\infty} h_n(r) z^{n-1} = \frac{a + (b - ap)rz}{1 - prz - qz^2}. \tag{1.4}$$

2. Main results

We begin this section by defining the subclass $\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$ as follows:

Definition 1. For $\delta \geq 0$, $0 \leq \lambda \leq 1$, $0 \leq \theta < 1$ and $r \in \mathbb{R}$, a function $f \in \Sigma$ is said to be in the class $\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$ if it satisfies the subordinations

$$(1-\delta)\left[(1-\lambda)\frac{z\,(f*I_\theta)'(z)}{(f*I_\theta)(z)} + \lambda\left(1 + \frac{z\,(f*I_\theta)''(z)}{(f*I_\theta)'(z)}\right)\right] + \delta\frac{\lambda z^2\,(f*I_\theta)''(z) + z\,(f*I_\theta)'(z)}{\lambda z\,(f*I_\theta)'(z) + (1-\lambda)\,(f*I_\theta)(z)} \prec \Pi(r,z) + 1 - a$$

and

$$(1-\delta)\left[(1-\lambda)\frac{w\,(g*I_\theta)'(w)}{(g*I_\theta)(w)} + \lambda\left(1 + \frac{w\,(g*I_\theta)''(w)}{(g*I_\theta)'(w)}\right)\right]$$

$$+\delta\frac{\lambda w^2(g*I_\theta)''(w)+w(g*I_\theta)'(w)}{\lambda w(g*I_\theta)'(w)+(1-\lambda)(g*I_\theta)(w)}\prec\Pi(r,w)+1-a,$$

where a is a real constant and the function $g=f^{-1}$ is given by (1.2).

Theorem 1. *For $\delta\geq 0$, $0\leq\lambda\leq 1$, $0\leq\theta<1$ and $r\in\mathbb{R}$, let $f\in\mathcal{A}$ be in the class $\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$. Then*

$$|a_2|\leq\frac{|br|\sqrt{|br|}}{\sqrt{2\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p(1-\theta)^2(\lambda+1)^2\right]br^2-2qa(1-\theta)^2(\lambda+1)^2\right|}}$$

and

$$|a_3|\leq\frac{|br|}{2(1-\theta)(3-2\theta)(2\lambda+1)}+\frac{b^2r^2}{4(1-\theta)^2(\lambda+1)^2}.$$

Proof. Let $f\in\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$. Then there are two analytic functions $u,v:U\longrightarrow U$ given by

$$u(z)=u_1z+u_2z^2+u_3z^3+\cdots\quad(z\in U)\tag{2.1}$$

and

$$v(w)=v_1w+v_2w^2+v_3w^3+\cdots\quad(w\in U),\tag{2.2}$$

with $u(0)=v(0)=0$, $|u(z)|<1$, $|v(w)|<1$, $z,w\in U$ such that

$$(1-\delta)\left[(1-\lambda)\frac{z(f*I_\theta)'(z)}{(f*I_\theta)(z)}+\lambda\left(1+\frac{z(f*I_\theta)''(z)}{(f*I_\theta)'(z)}\right)\right]$$
$$+\delta\frac{\lambda z^2(f*I_\theta)''(z)+z(f*I_\theta)'(z)}{\lambda z(f*I_\theta)'(z)+(1-\lambda)(f*I_\theta)(z)}=\Pi(r,u(z))+1-a$$

and

$$(1-\delta)\left[(1-\lambda)\frac{w(g*I_\theta)'(w)}{(g*I_\theta)(w)}+\lambda\left(1+\frac{w(g*I_\theta)''(w)}{(g*I_\theta)'(w)}\right)\right]$$
$$+\delta\frac{\lambda w^2(g*I_\theta)''(w)+w(g*I_\theta)'(w)}{\lambda w(g*I_\theta)'(w)+(1-\lambda)(g*I_\theta)(w)}=\Pi(r,v(w))+1-a.$$

Or, equivalently

$$(1-\delta)\left[(1-\lambda)\frac{z(f*I_\theta)'(z)}{(f*I_\theta)(z)}+\lambda\left(1+\frac{z(f*I_\theta)''(z)}{(f*I_\theta)'(z)}\right)\right]$$
$$+\delta\frac{\lambda z^2(f*I_\theta)''(z)+z(f*I_\theta)'(z)}{\lambda z(f*I_\theta)'(z)+(1-\lambda)(f*I_\theta)(z)}=1+h_1(r)+h_2(r)u(z)+h_3(r)u^2(z)+\cdots\tag{2.3}$$

and

$$(1-\delta)\left[(1-\lambda)\frac{w(g*I_\theta)'(w)}{(g*I_\theta)(w)}+\lambda\left(1+\frac{w(g*I_\theta)''(w)}{(g*I_\theta)'(w)}\right)\right]$$

$$+\delta\frac{\lambda w^2\left(g*I_\theta\right)''(w)+w\left(g*I_\theta\right)'(w)}{\lambda w\left(g*I_\theta\right)'(w)+(1-\lambda)\left(g*I_\theta\right)(w)}=1+h_1(r)+h_2(r)v(w)+h_3(r)v^2(w)+\cdots. \tag{2.4}$$

Combining (2.1), (2.2), (2.3) and (2.4) yields

$$\begin{aligned}&(1-\delta)\left[(1-\lambda)\frac{z\left(f*I_\theta\right)'(z)}{\left(f*I_\theta\right)(z)}+\lambda\left(1+\frac{z\left(f*I_\theta\right)''(z)}{\left(f*I_\theta\right)'(z)}\right)\right]\\&+\delta\frac{\lambda z^2\left(f*I_\theta\right)''(z)+z\left(f*I_\theta\right)'(z)}{\lambda z\left(f*I_\theta\right)'(z)+(1-\lambda)\left(f*I_\theta\right)(z)}=1+h_2(r)u_1z+\left[h_2(r)u_2+h_3(r)u_1^2\right]z^2+\cdots\end{aligned} \tag{2.5}$$

and

$$\begin{aligned}&(1-\delta)\left[(1-\lambda)\frac{w\left(g*I_\theta\right)'(w)}{\left(g*I_\theta\right)(w)}+\lambda\left(1+\frac{w\left(g*I_\theta\right)''(w)}{\left(g*I_\theta\right)'(w)}\right)\right]\\&+\delta\frac{\lambda w^2\left(g*I_\theta\right)''(w)+w\left(g*I_\theta\right)'(w)}{\lambda w\left(g*I_\theta\right)'(w)+(1-\lambda)\left(g*I_\theta\right)(w)}=1+h_2(r)v_1w+\left[h_2(r)v_2+h_3(r)v_1^2\right]w^2+\cdots.\end{aligned} \tag{2.6}$$

It is quite well-known that if $|u(z)|<1$ and $|v(w)|<1$, $z,w\in U$, then

$$|u_i|\leq 1 \quad and \quad |v_i|\leq 1 \; forall \; i\in\mathbb{N}. \tag{2.7}$$

Comparing the corresponding coefficients in (2.5) and (2.6), after simplifying, we have

$$2(1-\theta)(\lambda+1)a_2=h_2(r)u_1, \tag{2.8}$$

$$\begin{aligned}2(1-\theta)(3-2\theta)(2\lambda+1)a_3-4\left(1-\theta\right)^2\left(\lambda\delta(\lambda-1)+3\lambda+1\right)a_2^2\\=h_2(r)u_2+h_3(r)u_1^2,\end{aligned} \tag{2.9}$$

$$-2(1-\theta)(\lambda+1)a_2=h_2(r)v_1 \tag{2.10}$$

and

$$\begin{aligned}2(1-\theta)(3-2\theta)(2\lambda+1)(2a_2^2-a_3)-4\left(1-\theta\right)^2\left(\lambda\delta(\lambda-1)+3\lambda+1\right)a_2^2\\=h_2(r)v_2+h_3(r)v_1^2.\end{aligned} \tag{2.11}$$

It follows from (2.8) and (2.10) that

$$u_1=-v_1 \tag{2.12}$$

and

$$8\left(1-\theta\right)^2\left(\lambda+1\right)^2a_2^2=h_2^2(r)(u_1^2+v_1^2). \tag{2.13}$$

If we add (2.9) to (2.11), we find that

$$4(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]a_2^2=h_2(r)(u_2+v_2)+h_3(r)(u_1^2+v_1^2). \tag{2.14}$$

Substituting the value of $u_1^2+v_1^2$ from (2.13) in the right hand side of (2.14), we deduce that

$$a_2^2=\frac{h_2^3(r)(u_2+v_2)}{4\left[h_2^2(r)(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]-2h_3(r)\left(1-\theta\right)^2\left(\lambda+1\right)^2\right]}. \tag{2.15}$$

If we make further computations using (1.3), (2.7) and (2.15), we obtain

$$|a_2|\le\frac{|br|\sqrt{|br|}}{\sqrt{2\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p\left(1-\theta\right)^2\left(\lambda+1\right)^2\right]br^2-2qa\left(1-\theta\right)^2\left(\lambda+1\right)^2\right|}}.$$

Next, if we subtract (2.11) from (2.9), we can easily see that

$$4(1-\theta)(3-2\theta)(2\lambda+1)(a_3-a_2^2)=h_2(r)(u_2-v_2)+h_3(r)(u_1^2-v_1^2). \tag{2.16}$$

In view of (2.12) and (2.13), we get from (2.16)

$$a_3=\frac{h_2(r)(u_2-v_2)}{4(1-\theta)(3-2\theta)(2\lambda+1)}+\frac{h_2^2(r)(u_1^2+v_1^2)}{8\left(1-\theta\right)^2\left(\lambda+1\right)^2}.$$

Thus applying (1.3), we obtain

$$|a_3|\le\frac{|br|}{2(1-\theta)(3-2\theta)(2\lambda+1)}+\frac{b^2r^2}{4\left(1-\theta\right)^2\left(\lambda+1\right)^2}.$$

This completes the proof of Theorem 1. □

In the next theorem, we discuss the Fekete-Szegő problem for the subclass $\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$.

Theorem 2. *For $\delta\ge 0$, $0\le\lambda\le 1$, $0\le\theta<1$ and $r,\mu\in\mathbb{R}$, let $f\in\mathcal{A}$ be in the class $\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$. Then* $\left|a_3-\mu a_2^2\right|\le$

$$\le\begin{cases}\dfrac{|br|}{2(1-\theta)(3-2\theta)(2\lambda+1)};\\ \quad for\ |\mu-1|\le\dfrac{\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p\left(1-\theta\right)^2\left(\lambda+1\right)^2\right]br^2-2qa\left(1-\theta\right)^2\left(\lambda+1\right)^2\right|}{b^2r^2(1-\theta)(3-2\theta)(2\lambda+1)},\\ \dfrac{|br|^3\,|\mu-1|}{\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p\left(1-\theta\right)^2\left(\lambda+1\right)^2\right]br^2-2qa\left(1-\theta\right)^2\left(\lambda+1\right)^2\right|};\\ \quad for\ |\mu-1|\ge\dfrac{\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p\left(1-\theta\right)^2\left(\lambda+1\right)^2\right]br^2-2qa\left(1-\theta\right)^2\left(\lambda+1\right)^2\right|}{b^2r^2(1-\theta)(3-2\theta)(2\lambda+1)}.\end{cases}$$

Proof. It follows from (2.15) and (2.16) that

$$\begin{aligned}a_3-\mu a_2^2&=\frac{h_2(r)(u_2-v_2)}{4(1-\theta)(3-2\theta)(2\lambda+1)}+(1-\mu)\,a_2^2\\&=\frac{h_2(r)(u_2-v_2)}{4(1-\theta)(3-2\theta)(2\lambda+1)}\end{aligned}$$

$$+\frac{h_2^3(r)(u_2+v_2)(1-\mu)}{4\left[h_2^2(r)(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]-2h_3(r)(1-\theta)^2(\lambda+1)^2\right]}$$

$$=\frac{h_2(r)}{4}\left[\left(\psi(\mu,r)+\frac{1}{(1-\theta)(3-2\theta)(2\lambda+1)}\right)u_2\right.$$

$$\left.+\left(\psi(\mu,r)-\frac{1}{(1-\theta)(3-2\theta)(2\lambda+1)}\right)v_2\right],$$

where

$$\psi(\mu,r)=\frac{h_2^2(r)(1-\mu)}{h_2^2(r)(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]-2h_3(r)(1-\theta)^2(\lambda+1)^2}.$$

According to (1.3), we find that

$$\left|a_3-\mu a_2^2\right|\leq\begin{cases}\dfrac{|br|}{2(1-\theta)(3-2\theta)(2\lambda+1)}, & 0\leq|\psi(\mu,r)|\leq\dfrac{1}{(1-\theta)(3-2\theta)(2\lambda+1)},\\[2ex] \dfrac{1}{2}|br|\,|\psi(\mu,r)|, & |\psi(\mu,r)|\geq\dfrac{1}{(1-\theta)(3-2\theta)(2\lambda+1)}.\end{cases}$$

After some computations, we obtain $\left|a_3-\mu a_2^2\right|\leq$

$$\leq\begin{cases}\dfrac{|br|}{2(1-\theta)(3-2\theta)(2\lambda+1)};\ for\ |\mu-1|\leq\\[2ex] \leq\dfrac{\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p(1-\theta)^2(\lambda+1)^2\right]br^2-2qa(1-\theta)^2(\lambda+1)^2\right|}{b^2r^2(1-\theta)(3-2\theta)(2\lambda+1)},\\[2ex] \dfrac{|br|^3|\mu-1|}{\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p(1-\theta)^2(\lambda+1)^2\right]br^2-2qa(1-\theta)^2(\lambda+1)^2\right|};\\[2ex] for\ |\mu-1|\geq\\[2ex] \geq\dfrac{\left|\left[(1-\theta)\left[2\lambda\delta(1-\theta)(1-\lambda)+2\theta\lambda+1\right]b-2p(1-\theta)^2(\lambda+1)^2\right]br^2-2qa(1-\theta)^2(\lambda+1)^2\right|}{b^2r^2(1-\theta)(3-2\theta)(2\lambda+1)}.\end{cases}$$

□

Putting $\mu=1$ in Theorem 2, we obtain the following result:

Corollary 1. *For $\delta\geq 0$, $0\leq\lambda\leq 1$, $0\leq\theta<1$ and $r\in\mathbb{R}$, let $f\in\mathcal{A}$ be in the class $\mathcal{N}_\Sigma(\delta,\lambda,\theta,r)$. Then*

$$\left|a_3-a_2^2\right|\leq\frac{|br|}{2(1-\theta)(3-2\theta)(2\lambda+1)}.$$

Remark 2. Special cases are shown below:

(1) If we put $\lambda = 0$ and $\theta = \frac{1}{2}$ in our Theorems, we have the results for well-known class $S^*_\Sigma(r)$ of bi-starlike functions which was studied recently by Srivastava et al. [16].
(2) If we put $\lambda = 1$ and $\theta = \frac{1}{2}$ in our Theorems, we have the results for the class $\mathcal{K}_\Sigma(r)$ which was considered recently by Magesh et al. [12].
(3) If we put $\delta = 0$ and $\theta = \frac{1}{2}$ in our Theorems, we have the results for the class $M_\Sigma(\lambda, r)$ which was investigated recently by Magesh et al. [12].
(4) If we put $\lambda = 0$, $\theta = \frac{1}{2}$, $a = 1$, $b = p = 2$, $q = -1$ and $r \longrightarrow t$ in our Theorems, we obtain the results for the class $S^*_\Sigma(t)$ of bi-starlike functions which was introduced recently by Altınkaya and Yalçin [3].
(5) If we put $\delta = 0$, $\theta = \frac{1}{2}$, $a = 1$, $b = p = 2$, $q = -1$ and $r \longrightarrow t$ in our Theorems, we have the results for the class $\mathcal{M}^*_\Sigma(\alpha, t)$ which was considered recently by Altınkaya and Yalçin [2].

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