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FUZZY APPROXIMATION OF AN ADDITIVE (ρ_1, ρ_2)-RANDOM OPERATOR INEQUALITY

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Abstract. We study and solve an additive (ρ_1, ρ_2)-random operator inequality in which $\rho_1, \rho_2 \in \mathbb{C}$ are fixed and $\max\{\sqrt{2}|\rho_1|,\ |\rho_2|\} < 1$.

Finally, we get a fuzzy approximation of the mentioned additive (ρ_1, ρ_2)-random operator inequality.

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1. Introduction

Let $(\Omega, \mathfrak{U}, \mu)$ be a probability measure space. Assume that $(U, \mathfrak{B}_U)$ and $(V, \mathfrak{B}_V)$ are Borel measureable spaces, in which U and V are complete *fuzzy normed spaces* (in short, *FN-spaces*) and $T : \Omega \times U \to V$ is a random operator. In FN-spaces, first we solve the (ρ_1, ρ_2)-random operator inequality

$$\begin{aligned} &\eta(T(\omega, u+v) - T(\omega, u) - T(\omega, v), t) \\ &\geq \kappa_M \Bigg(\eta(\rho_1(T(\omega, u+v) + T(\omega, u-v) - 2T(\omega, u)), t), \\ &\qquad \eta\left(\rho_2\left(2T\left(\omega, \frac{u+v}{2}\right) - T(\omega, u) - T(\omega, v)\right), t\right)\Bigg), \end{aligned} \tag{1.1}$$

in which $\rho_1, \rho_2 \in \mathbb{C}$ are fixed and $\max\{\sqrt{2}|\rho_1|,\ |\rho_2|\} < 1$.

Next, we get a fuzzy approximation of the (ρ_1, ρ_2)-random operator inequality (1.1). Fuzzy approximation of additive random operator inequality has some applications for generating secret keys in client–server environment [11].

2. Preliminaries

In this paper, we let $I = [0, 1]$ and $J = (0, 1]$.

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Definition 1 ([7, Page 15] and [17, Definition 5.5.1])**.** A *continuous triangular norm* (shortly, a *ct-norm*) is a continuous mapping κ from I^2 to I such that

(a) $\kappa(\tau,\upsilon)=\kappa(\upsilon,\tau)$ and $\kappa(\tau,\kappa(\upsilon,\vartheta))=\kappa(\kappa(\tau,\upsilon),\vartheta)$ for all $\tau,\upsilon,\vartheta\in I$;
(b) $\kappa(\tau,1)=\tau$ for all $\tau\in I$;
(c) $\kappa(\tau,\upsilon)\le\kappa(\vartheta,\iota)$ whenever $\tau\le\vartheta$ and $\upsilon\le\iota$ for all $\tau,\upsilon,\vartheta,\iota\in I$.

Some examples of the t-norms are:

(1) $\kappa_P(\tau,\upsilon)=\tau\upsilon$;
(2) $\kappa_M(\tau,\upsilon)=\min\{\tau,\upsilon\}$;
(3) $\kappa_L(\tau,\upsilon)=\max\{\tau+\upsilon-1,0\}$ (the Lukasiewicz t-norm).

Definition 2 ([16, Definition 2] and [2, Definition 2.4])**.** Suppose that κ is a ct-norm, V is a linear space and η is a fuzzy set from V to J. In this case, the ordered tuple (V,η,κ) is called a FN-space if the following conditions are satisfied:

(FN1) $\eta(v,t)=1$ for all $t>0$ if and only if $v=0$;
(FN2) $\eta(\alpha v,t)=\eta\left(v,\frac{t}{|\alpha|}\right)$ for all $v\in V$ and $\alpha\in\mathbb{C}$ with $\alpha\neq 0$;
(FN3) $\eta(u+v,t+s)\ge\kappa(\eta(u,t),\eta(v,s))$ for all $u,v\in V$ and $t,s\ge 0$.
(FN4) $\eta(u,.):(0,\infty)\to J$ is continuous.

Let $(V,\|\cdot\|)$ be a linear normed space. Then

$$\eta(v,s)=\exp\left(-\frac{\|v\|}{s}\right)$$

for all $s>0$ defines a fuzzy norm and the ordered tuple (V,η,κ_M) is a FN-space.

Let $(\Omega,\mathfrak{U},\mu)$ be a probability measure space. Assume that $(U,\mathfrak{B}_U)$ and $(V,\mathfrak{B}_V)$ are Borel measureable spaces, in which U and V are complete FN-spaces. A mapping $T\colon\Omega\times U\to V$ is said to be a random operator if $\{\omega:\ T(\omega,u)\in B\}\in\mathfrak{U}$ for all u in U and $B\in\mathfrak{B}_V$. Also, T is random operator, if $T(\omega,u)=v(\omega)$ be a V-valued random variable for every u in U. A random operator $T:\Omega\times U\to V$ is called *linear* if $T(\omega,\alpha u_1+\beta u_2)=\alpha T(\omega,u_1)+\beta T(\omega,u_2)$ almost everywhere for each u_1,u_2 in U and scalers $\alpha,\beta\in\mathbb{C}$, and *bounded* if there exists a nonnegative real-valued random variable $M(\omega)$ such that

$$\eta(T(\omega,u_1)-T(\omega,u_2),M(\omega)t)\ge\eta(u_1-u_2,t),$$

almost everywhere for each u_1,u_2 in U and $t>0$.

Theorem 1 ([3, Proposition 1.3] and [5, Page 306])**.** *Consider a complete generalized metric space* (Γ,Δ) *and a strictly contractive function* $L\colon\Gamma\to\Gamma$ *with Lipschitz constant* $\beta<1$. *Then, for every given element* $\gamma\in\Gamma$*, either*

$$\Delta\left(L^m\gamma,L^{m+1}\gamma\right)=\infty$$

for each $m\in\mathbb{N}$ *or there is* $m_0\in\mathbb{N}$ *such that*

(1) $\Delta\left(L^m\gamma,L^{m+1}\gamma\right)<\infty,\ \forall m\ge m_0$*;*

(2) the fixed point ϖ^* of L is the limit of sequence $\{L^m\gamma\}$;
(3) in the set $\Upsilon = \{\varpi \in \Gamma \mid \Delta(L^{m_0}\gamma,\varpi) < \infty\}$, ϖ^* is the unique fixed point of L;
(4) $(1-\beta)\Delta(\varpi,\varpi^*) \leq \Delta(\varpi,L\varpi)$ for every $\varpi \in \Upsilon$.

Recently, some authors have published several papers on approximation of functional equations in different spaces by the direct technique and the fixed point technique, for example, fuzzy Menger normed algebras [10], fuzzy metric spaces [15], fuzzy normed spaces [13], non-Archimedean random Lie C^*-algebras [8], non-Archimedean random normed spaces [18], random multi-normed space [1], see also [4], [6] and [14].

3. Approximation of additive (ρ_1,ρ_2)-random operator inequality

Now, we are ready to get a fuzzy approximation of the (ρ_1,ρ_2)-random operator inequality (1.1) as a generalization of Park's results [12].

Lemma 1. *Assume that (V,η,κ_M) is a FN-space. Let $T:\Omega\times U\to V$ be a random operator, where $T(\omega,0)=0$ and satisfies (1.1), then T is additive.*

Proof. Replacing v by u in (1.1) and using (FN2), we get

$$\eta(T(\omega,2u)-2T(\omega,u),t) \geq \eta\left(T(\omega,2u)-2T(\omega,u),\frac{t}{|\rho_1|}\right).$$

Since $|\rho_1|<1$, $T(\omega,2u)=2T(\omega,u)$ for each $u\in U$ and so

$$T\left(\omega,\frac{u}{2}\right)=\frac{1}{2}T(\omega,u) \tag{3.1}$$

almost everywhere for each $u\in U$ and $\omega\in\Omega$.

Using (1.1) and (3.1) imply that

$$\begin{aligned}&\eta(T(\omega,u+v)-T(\omega,u)-T(\omega,v),t)\\&\geq \kappa_M\left(\eta\left(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u),\frac{t}{|\rho_1|}\right),\right.\\&\qquad\left.\eta\left(T(\omega,u+v)-T(\omega,u)-T(\omega,v),\frac{t}{|\rho_2|}\right)\right),\end{aligned}$$

and so

$$\begin{aligned}&\eta(T(\omega,u+v)-T(\omega,u)-T(\omega,v),t)\\&\geq \eta\left(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u),\frac{t}{|\rho_1|}\right)\end{aligned} \tag{3.2}$$

almost everywhere for each $u,v\in U$, $\omega\in\Omega$ and $t>0$.

Putting $z = u+v$ and $w = u-v$ in (3.2), we get

$$\begin{aligned}&\eta(T(\omega,z+w)+T(\omega,z-w)-2T(\omega,z),t)\\&\geq \eta\left(T(\omega,z+w)-T(\omega,z)-T(\omega,w),\frac{t}{2|\rho_1|}\right)\end{aligned} \tag{3.3}$$

almost everywhere for every $z,w \in U$ and $\omega \in \Omega$.

Now, (3.2), (3.3) and (FN2) imply that

$$\begin{aligned}&\eta(T(\omega,u+v)-T(\omega,u)-T(\omega,v),t)\\&\geq \eta\left(T(\omega,u+v)-T(\omega,u)-T(\omega,v),\frac{t}{2|\rho_1|^2}\right)\end{aligned}$$

almost everywhere for all $u,v \in U$, $\omega \in \Omega$ and $t>0$. Since $|\rho_1| < \frac{\sqrt{2}}{2}$, $T(\omega,u+v) = T(\omega,u)+T(\omega,v)$ almost everywhere for all $u,v \in U$ and $\omega \in \Omega$, which implies that T is additive. □

We use fixed point technique to get an approximation of the additive (ρ_1,ρ_2)-random operator inequality (1.1) in FN-space.

Theorem 2. *Consider the complete FN-space (V,η,κ_M). Assume that $\psi\colon U^2\times(0,\infty)\to J$ is a fuzzy set such that there exists an $\beta<1$ with*

$$\psi\left(\frac{u}{2},\frac{v}{2},\frac{\beta t}{2}\right) \geq \psi(u,v,t) \qquad \text{and} \qquad \lim_{p\to\infty}\psi\left(\frac{u}{2^p},\frac{v}{2^p},\frac{t}{2^p}\right)=1$$

for each $u,v\in U$ and $t>0$. Suppose that $T\colon \Omega\times U\to V$ is a random operator, where $T(\omega,0)=0$ and

$$\begin{aligned}&\eta(T(\omega,u+v)-T(\omega,u)-T(\omega,v),t)\\&\geq \kappa_M\Bigg(\eta(\rho_1(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u)),t),\\&\qquad \eta\left(\rho_2\left(2T\left(\omega,\frac{u+v}{2}\right)-T(\omega,u)-T(\omega,v)\right),t\right),\psi(u,v,t)\Bigg),\end{aligned} \tag{3.4}$$

in which $\rho_1,\rho_2\in\mathbb{C}$ are fixed and $\max\{\sqrt{2}|\rho_1|,\ |\rho_2|\}<1$. Then, there is a unique additive random operator $S\colon \Omega\times U\to V$ such that

$$\eta(T(\omega,u)-S(\omega,u),t) \geq \psi\left(u,u,\frac{2(1-\beta)}{\beta}t\right)$$

almost everywhere for each $u\in U$, $\omega\in\Omega$ and $t>0$.

Proof. Putting $u=v$ in (3.4), implies that

$$\eta\left(2T\left(\omega,\frac{u}{2}\right)-T(\omega,u),t\right) \geq \psi\left(\frac{u}{2},\frac{u}{2},t\right) \tag{3.5}$$

almost everywhere for each $u\in U$, $\omega\in\Omega$ and $t>0$.

On

$$\Gamma := \{H\colon \Omega\times U \to V,\ \ H(\omega,0)=0\}$$

we define the following generalized metric:

$$\Delta(G,H)=\inf\{\alpha\in\mathbb{R}_+ : \eta(G(\omega,u)-H(\omega,u),\alpha t)\geq \psi(u,u,t),\ \ \forall u\in U,\ \ t>0\}.$$

In [9], Mihet and Radu proved that (Γ,Δ) is a complete generalized metric.

We define the linear function $L\colon \Gamma\to\Gamma$ as

$$LG(\omega,u):=2G\left(\omega,\frac{u}{2}\right)$$

almost everywhere for each $u\in U$ and $\omega\in\Omega$. Consider $G,H\in\Gamma$ such that $\Delta(G,H)=\varepsilon$. Then,

$$\eta(G(\omega,u)-H(\omega,u),\varepsilon t)\geq\psi(u,u,t)$$

almost everywhere for each $u\in U$, $\omega\in\Omega$ and $t>0$. Also,

$$\begin{aligned}\eta(LG(\omega,u)-LH(\omega,u),\beta\varepsilon t)&=\eta\left(G\left(\omega,\frac{u}{2}\right)-H\left(\omega,\frac{u}{2}\right),\frac{\beta\varepsilon t}{2}\right)\\&\geq\psi\left(\frac{u}{2},\frac{u}{2},\frac{\beta t}{2}\right)\geq\psi(u,u,t)\end{aligned}$$

almost everywhere for each $u\in U$, $\omega\in\Omega$ and $t>0$. Then, from $\Delta(G,H)=\varepsilon$ we conclude that $\Delta(LG,LH)\leq\beta\varepsilon$ and so

$$\Delta(LG,LH)\leq\beta\Delta(G,H)$$

for each $G,H\in\Gamma$.

By (3.5) we have that

$$\eta\left(2T\left(\omega,\frac{u}{2}\right)-T(\omega,u),\frac{\beta t}{2}\right)\geq\psi(u,u,t)$$

almost everywhere for each $u\in U$ and $t>0$, which implies that $\Delta(T,LT)\leq\frac{\beta}{2}$.

Theorem 1 implies that, there exists a random operator $S\colon \Omega\times U\to V$ such that:

(1) A fixed point for function L, is S,

$$S(\omega,u)=2S\left(\omega,\frac{u}{2}\right)$$

almost everywhere for each $u\in U$ and $\omega\in\Omega$, which is unique in the set

$$\Upsilon=\{G\in\Gamma:\ \Delta(G,H)<\infty\};$$

(2) $\Delta(L^pT,S)\to 0$ as $p\to\infty$, which implies that

$$\lim_{p\to\infty}2^pT\left(\omega,\frac{u}{2^p}\right)=S(\omega,u) \tag{3.6}$$

almost everywhere for each $u\in U$ and $\omega\in\Omega$;

(3) $\Delta(T,S) \leq \frac{1}{1-\beta}\Delta(T,LT)$, which implies
$$\eta(T(\omega,u)-S(\omega,u),t) \geq \psi\left(u,u,\frac{2(1-\beta)}{\beta}t\right)$$
almost everywhere for each $u \in U$, $\omega \in \Omega$ and $t > 0$.

Using (3.4) and (3.6) imply that
$$\begin{aligned}
&\eta(S(\omega,u+v)-S(\omega,u)-S(\omega,v),t)\\
&= \lim_{p\to\infty}\eta\left(T\left(\omega,\frac{u+v}{2^p}\right)-T\left(\omega,\frac{u}{2^p}\right)-T\left(\omega,\frac{v}{2^p}\right),\frac{t}{2^p}\right)\\
&\geq \lim_{p\to\infty}\kappa_M\left(\eta\left(\rho_1\left(T\left(\omega,\frac{u+v}{2^p}\right)+T\left(\omega,\frac{u-v}{2^p}\right)-2T\left(\omega,\frac{u}{2^p}\right)\right),\frac{t}{2^p}\right),\right.\\
&\qquad \eta\left(\rho_2\left(2T\left(\omega,\frac{u+v}{2^{p+1}}\right)-T(\omega,\frac{u}{2^p})-T(\omega,\frac{v}{2^p})\right),\frac{t}{2^p}\right),\\
&\qquad \left.\psi\left(\frac{u}{2^p},\frac{v}{2^p},\frac{t}{2^p}\right)\right)\\
&= \kappa_M\left(\eta(\rho_1(S(\omega,u+v)+S(\omega,u-v)-2S(\omega,u)),t),\right.\\
&\qquad \left.\eta\left(\rho_2\left(2S\left(\omega,\frac{u+v}{2}\right)-S(\omega,u)-S(\omega,v)\right),t\right)\right)2
\end{aligned}$$
almost everywhere for each $u,v \in U$, $\omega \in \Omega$ and $t > 0$. Then
$$\begin{aligned}
&\eta(S(\omega,u+v)-S(\omega,u)-S(\omega,v),t)\\
&\geq \kappa_M\left(\eta(\rho_1(S(\omega,u+v)+S(\omega,u-v)-2S(\omega,u)),t),\right.\\
&\qquad \left.\eta\left(\rho_2\left(2S\left(\omega,\frac{u+v}{2}\right)-S(\omega,u)-S(\omega,v)\right),t\right)\right)
\end{aligned}$$
almost everywhere for each $u,v \in U$, $\omega \in \Omega$ and $t > 0$. Now, Lemma 1, implies that S is an additive random operator. □

Corollary 1. *Let (V,η,κ_M) be a FN-space, $\rho > 1$ and $\tau > 0$. Suppose that $T : \Omega \times U \to V$ is a random operator, where $T(\omega,0) = 0$ and*
$$\begin{aligned}
&\eta(T(\omega,u+v)-T(\omega,u)-T(\omega,v),t)\\
&\geq \kappa_M\left(\eta(\rho_1(T(\omega,u+v)+T(\omega,u-v)-2T(\omega,u)),t),\right.\\
&\qquad \eta\left(\rho_2\left(2T\left(\omega,\frac{u+v}{2}\right)-T(\omega,u)-T(\omega,v)\right),t\right),\\
&\qquad \left.\frac{t}{t+\tau(\|u\|^\rho+\|v\|^\rho)}\right),
\end{aligned} \tag{3.7}$$

in which $\rho_1,\rho_2 \in \mathbb{C}$ *are fixed and* $\max\{\sqrt{2}|\rho_1|,\ |\rho_2|\} < 1$. *Then, there is a unique additive random operator* $S\colon \Omega\times U \to V$ *such that*

$$\eta(T(\omega,u)-S(\omega,u),t) \geq \frac{(2^{\rho-1}-1)t}{(2^{\rho-1}-1)t+\tau\|u\|^{\rho}}$$

for each $u\in U$, $\omega\in\Omega$ *and* $t>0$.

Proof. In Theorem 2, we put $\psi(u,v,t) = \frac{t}{t+\tau(\|u\|^{\rho}+\|v\|^{\rho})}$ for each $u,v\in U$, $t>0$ and $\beta = 2^{1-\rho}$. Now, we show that the fuzzy control function ψ satisfies in the conditions of Theorem 2.

Let $u,v\in U$, $t>0$ and $\beta=2^{1-\rho}$. Then we have

$$\begin{aligned}\psi\left(\frac{u}{2},\frac{v}{2},\frac{\beta t}{2}\right) &= \frac{2^{-\rho}t}{2^{-\rho}t+\tau(\|\frac{u}{2}\|^{\rho}+\|\frac{v}{2}\|^{\rho})} = \frac{2^{-\rho}t}{2^{-\rho}t+\frac{1}{2^{\rho}}\tau(\|u\|^{\rho}+\|u\|^{\rho})}\\ &= \frac{t}{t+\tau(\|u\|^{\rho}+\|v\|^{\rho})} = \psi(u,v,t),\end{aligned}$$

also

$$\begin{aligned}\lim_{n\to\infty}\psi\left(\frac{u}{2^n},\frac{v}{2^n},\frac{t}{2^n}\right) &= \frac{2^{1-\rho}\frac{t}{2^n}}{2^{1-\rho}\frac{t}{2^n}+\tau(\|\frac{u}{2^n}\|^{\rho}+\|\frac{v}{2^n}\|^{\rho})}\\ &= \lim_{n\to\infty}\frac{2^{1-\rho}2^{n(\rho-1)}t}{2^{1-\rho}2^{n(\rho-1)}t+\tau(\|u\|^{\rho}+\|v\|^{\rho})}\\ &= \lim_{n\to\infty}\frac{2^{1-\rho}2^{n(\rho-1)}t}{2^{1-\rho}2^{n(\rho-1)}t} = 1.\end{aligned}$$

□

Theorem 3. *Consider the complete FN-space* (V,η,κ_M). *Suppose that* $\psi\colon U^2\times(0,\infty)\to J$ *is a fuzzy set such that there exists an* $\beta<1$ *with*

$$\psi(2u,2v,2\beta t)\geq\psi(u,v,t) \qquad \text{and} \qquad \lim_{p\to\infty}\psi(2^p u,2^p v,2^p t)=1$$

for all $u,v\in U$ *and* $t>0$. *Suppose that* $T\colon \Omega\times U\to V$ *be a random operator in which* $T(\omega,0)=0$ *and satisfies in* (3.4). *So, there is a unique additive random operator* $S\colon \Omega\times U\to V$ *such that*

$$\eta(T(\omega,u)-S(\omega,u),t)\geq\psi(u,u,2(1-\beta)t)$$

almost everywhere for each $u\in U$ *and* $t>0$.

Proof. Let (Γ,Δ) be same as proof of Theorem 2. We define the linear function $L:\Gamma\to\Gamma$ as

$$LG(\omega,u) := \frac{1}{2}G(\omega,2u)$$

almost everywhere for each $u \in U$. Using (3.5) implies that

$$\eta\left(\frac{T(\omega,2u)}{2} - T(\omega,u), t\right) \geq \psi\left(\frac{u}{2}, \frac{u}{2}, \frac{t}{\beta}\right)$$

almost everywhere for each $u \in U$ and $t > 0$.

By similar method of proof last theorem, the proof will be completed. □

Corollary 2. *Let (V,η,κ_M) be a FN-space, $\rho < 1$ and $\tau > 0$. Suppose that $T\colon \Omega \times U \to V$ is a random operator in which $T(\omega,0) = 0$ and satisfies (3.7). Then there exists a unique additive random operator $S\colon \Omega \times U \to V$ such that*

$$\eta(T(\omega,u) - S(\omega,u),t) \geq \frac{(2-2^\rho)t}{(2-2^\rho)t + 2\tau\|u\|^\rho}$$

for each $u \in U$ and $t > 0$.

Proof. In Theorem 3, we put $\psi(u,v,t) = \frac{t}{t+\tau(\|u\|^\rho+\|v\|^\rho)}$ for each $u,v \in U$, $t > 0$ and $\beta = 2^{\rho-1}$. Now, we show that the fuzzy control function ψ satisfies in the conditions of Theorem 3.

Let $u,v \in U$, $t > 0$ and $\beta = 2^{\rho-1}$. Then we have

$$\begin{aligned}\psi(2u,2v,2\beta t) &= \frac{2^{\rho-1}2t}{2^{\rho-1}2t + \tau(\|2u\|^\rho + \|2v\|^\rho)} = \frac{2^\rho t}{2^\rho t + 2^\rho \tau(\|u\|^\rho + \|v\|^\rho)} \\ &= \frac{t}{t + \tau(\|u\|^\rho + \|v\|^\rho)} = \psi(u,v,t),\end{aligned}$$

also

$$\begin{aligned}\lim_{n\to\infty} \psi(2^n u, 2^n v, 2^n t) &= \frac{2^{\rho-1}2^n t}{2^{\rho-1}2^n t + \tau(\|2^n u\|^\rho + \|2^n v\|^\rho)} \\ &= \lim_{n\to\infty} \frac{2^n 2^{\rho-1} t}{2^n 2^{\rho-1} t + 2^{n\rho}\tau(\|u\|^\rho + \|v\|^\rho)} \\ &= \lim_{n\to\infty} \frac{2^{n(1-\rho)} 2^{\rho-1} t}{2^{n(1-\rho)} 2^{\rho-1} t + \tau(\|u\|^\rho + \|v\|^\rho)} \\ &= \lim_{n\to\infty} \frac{2^{n(1-\rho)} 2^{\rho-1} t}{2^{n(\rho-1)} 2^{\rho-1} t} = 1.\end{aligned}$$

□

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