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ON WEAKLY SS-PERMUTABLE SUBGROUPS OF FINITE GROUPS

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Abstract. Suppose that G is a finite group and H is a subgroup of G. We say that: (1) H is ss-permutable in G if there is a subgroup B of G such that $G = HB$ and H permutes with every Sylow subgroup of B; (2) H is weakly ss-permutable in G if there are a subnormal subgroup T of G and an ss-permutable subgroup H_{ss} of G contained in H such that $G = HT$ and $H \cap T \leq H_{ss}$. We investigate the influence of weakly ss-permutable subgroups on the p-nilpotency and p-supersolvability of finite groups.

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1. INTRODUCTION

All groups considered in this paper are finite. A subgroup H of a group G is said to be s-permutable (or s-quasinormal) in G if H permutes with every Sylow subgroups of G [5]. In 2008, Shirong Li, etc. [7], introduced the concept of ss-permutability (or ss-quasinormality) which is a generalization of s-permutability. A subgroup H of a group G is called ss-permutable in G if there is a subgroup B of G such that $G = HB$ and H permutes with every Sylow subgroup of B. Li investigated the influence of ss-quasinormallity of some subgroups on the structure of finite groups. More recently, Xuanli He, etc. [3], introduced the following concept, which covers both ss-permutability and Skiba's weakly s-permutability [10] (A subgroup H of a group G is called weakly s-permutable in G if there is a subnormal subgroup T of G such that $G = HT$ and $H \cap T \leq H_{sG}$, where H_{sG} is the maximal s-permutable subgroup of G contained in H).

Definition 1. Let H be a subgroup of G. H is called weakly ss-permutable in G if there is a subnormal subgroup T of G such that $G = HT$ and $H \cap T \leq H_{ss}$, where H_{ss} is an ss-permutable subgroup of G contained in H

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In [3], Xuanli He studied the influence of weakly ss-permutable subgroups on the supersolvability of groups. In the present paper we characterize p-nilpotency and p-supersolvability of finite groups with the assumption that some maximal subgroups or 2-maximal subgroups are weakly ss-permutable.

2. Preliminaries

Lemma 1 ([3], Lemma 2.2)**.** *Let U be a weakly ss-permutable subgroup of a group G and N a normal subgroup of G. Then*

(a) *If $U \le H \le G$, then U is weakly ss-permutable in H.*

(b) *Suppose that U is a p-group for some prime p. If $N \le U$, then U/N is weakly ss-permutable in G/N.*

(c) *Suppose U is a p-group for some prime p and N is a p'-subgroup, then UN/N is weakly ss-permutable in G/N.*

(d) *Suppose U is a p-group for some prime p and U is not ss-permutable in G. Then G has a normal subgroup M such that $|G : M| = p$ and $G = UM$.*

(e) *If $U \le O_p(G)$ for some prime p, then U is weakly s-permutable in G.*

Lemma 2. *Let p be a prime dividing the order of a group G and P a Sylow p-subgroup of G. If $N_G(P)$ is p-nilpotent and P is abelian, then G is p-nilpotent.*

Proof. Since $N_G(P)$ is p-nilpotent, $N_G(P) = P \times H$, where H is the normal p-complement of $N_G(P)$. Since P is abelian and $[P, H] = 1$, we see that $C_G(P) = P \times H = N_G(P)$. Hence G is p-nilpotent. □

Lemma 3 ([1], A, 1.2)**.** *Let U, V, and W be subgroups of a group G. Then the following statements are equivalent:*

(1) $U \cap VW = (U \cap V)(U \cap W)$.

(2) $UV \cap UW = U(V \cap W)$.

Lemma 4 ([4], VI, 4.10)**.** *Assume that A and B are two subgroups of a group G and $G \ne AB$. If $AB^g = B^g A$ holds for any $g \in G$, then either A or B is contained in a nontrivial normal subgroup of G.*

Lemma 5 ([13], Lemma 3.16)**.** *Let $\mathcal{F}$ be the class of groups with Sylow tower of supersolvable type. Also let P be a normal p-subgroup of a group G such that $G/P \in \mathcal{F}$. If G is A_4-free and $|P| \leqslant p^2$, then $G \in \mathcal{F}$.*

Lemma 6 ([6], Lemma 2.6)**.** *Let H be a solvable normal subgroup of a group $G(H \ne 1)$. If every minimal normal subgroup of G which is contained in H is not contained in $\Phi(G)$, then the Fitting subgroup $F(H)$ of H is the direct product of minimal normal subgroups of G which are contained in H.*

Lemma 7 ([8], Lemma 2.2)**.** *If P is a s-permutable p-subgroup of G for some prime p, then $N_G(P) \ge O^p(G)$.*

Lemma 8 ([12], Lemma 2.8). *Let M be a maximal subgroup of G and P a normal p-subgroup of G such that $G = PM$, where p is a prime. Then $P \cap M$ is a normal subgroup of G.*

3. Results

Theorem 1. *Let P be a Sylow p-subgroup of a group G, where p is a prime divisor of $|G|$. If $N_G(P)$ is p-nilpotent and every maximal subgroups of P is weakly ss-permutable in G, then G is p-nilpotent.*

Proof. It is easy to see that the theorem holds when $p = 2$ by [3, Theorem 3.1], so it suffices to prove the theorem for the case of odd prime. Suppose that the theorem is false and let G be a counterexample of minimal order. We will derive a contradiction in several steps.

(1) G is not a non-abelian simple group.

By Lemma 2, $p^3 || P|$ and so there exists a non-identity maximal subgroup P_1 of P. By the hypothesis, P_1 is weakly ss-permutable in G. Then there are a subnormal subgroup T of G and an ss-permutable subgroup $(P_1)_{ss}$ of G contained in P_1 such that $G = P_1 T$ and $P_1 \cap T \leq (P_1)_{ss}$. Suppose G is simple, then $T = G$ and so $P_1 = (P_1)_{ss}$ is ss-permutable in G. By [7, Lemma 2.5] and Lemma 4, G has a nontrivial normal subgroup, a contradiction.

(2) $O_{p'}(G) = 1$.

If $O_{p'}(G) \neq 1$, we consider $G/O_{p'}(G)$. By Lemma 1, it is easy to see that every maximal subgroups of $PO_{p'}(G)/O_{p'}(G)$ is weakly ss-permutable in $G/O_{p'}(G)$. Since $N_{G/O_{p'}(G)}(PO_{p'}(G)/O_{p'}(G)) = N_G(P)O_{p'}(G)/O_{p'}(G)$ is p-nilpotent, $G/O_{p'}(G)$ satisfies all the hypotheses of our theorem. The minimality of G yields that $G/O_{p'}(G)$ is p-nilpotent, and so G is p-nilpotent, a contradiction.

(3) If M is a proper subgroup of G with $P \leq M < G$, then M is p-nilpotent.

It is clear to see $N_M(P) \leq N_G(P)$ and hence $N_M(P)$ is p-nilpotent. Applying Lemma 1, we immediately see that M satisfies the hypotheses of our theorem. Now, by the minimality of G, M is p-nilpotent.

(4) G has a unique minimal normal subgroup N such that G/N is p-nilpotent. Moreover $\Phi(G) = 1$.

Let N be a minimal normal subgroup of G. We shall prove that G/N satisfies the hypothesis of the theorem. Since P is a Sylow p-subgroup of G, PN/N is a Sylow p-subgroup of G/N. If $|PN/N| \leqslant p^2$, then G/N is p-nilpotent by Lemma 2. So we suppose $|PN/N| \geqslant p^3$. Let M_1/N be a maximal subgroup of PN/N. Then $M_1 = N(M_1 \cap P)$. Let $P_1 = M_1 \cap P$. It follows that $P_1 \cap N = M_1 \cap P \cap N = P \cap N$ is a Sylow p-subgroup of N. Since $p = |PN/N :$

$M_1/N| = |PN : (M_1 \cap P)N| = |P : M_1 \cap P| = |P : P_1|$, P_1 is a maximal subgroup of P. By the hypothesis, P_1 is weakly ss-permutable in G, then there are a subnormal subgroup T of G and an ss-permutable subgroup $(P_1)_{ss}$ of G contained in P_1 such that $G = P_1T$ and $P_1 \cap T \leq (P_1)_{ss}$. So $G/N = P_1N/N \cdot TN/N = M_1/N \cdot TN/N$. Since $(|N : P_1 \cap N|, |N : T \cap N|) = 1$, $(P_1 \cap N)(T \cap N) = N = N \cap G = N \cap (P_1T)$. By Lemma 3, $(P_1N) \cap (TN) = (P_1 \cap T)N$. It follows that $(P_1N/N) \cap (TN/N) = (P_1N \cap TN)/N = (P_1 \cap T)N/N \leq (P_1)_{ss}N/N$. Since $(P_1)_{ss}N/N$ is ss-permutable in G/N by [7, Lemma 2.1], M_1/N is weakly ss-permutable in G/N. Since $N_{G/N}(PN/N) = N_G(P)N/N$ is p-nilpotent, we have that G/N satisfies the hypothesis of the theorem. The choice of G yields that G/N is p-nilpotent. Consequently the uniqueness of N and the fact that $\Phi(G) = 1$ are obvious.

(5) $G = PQ$ is solvable, where Q is a Sylow q-subgroup of G with $p \neq q$.

Since G is not p-nilpotent, by a result of Thompson [11, Corollary], there exists a characteristic subgroup H of P such that $N_G(H)$ is not p-nilpotent. If $N_G(H) < G$, we must have $N_G(H)$ is p-nilpotent by step (3), a contradiction. We obtain $N_G(H) = G$. This leads to $O_p(G) \neq 1$. By step (4), $G/O_p(G)$ is p-nilpotent and therefore G is p-solvable. Then for any $q \in \pi(G)$ with $q \neq p$, there exists a Sylow q-subgroup of Q such that $G_1 = PQ$ is a subgroup of G [2, Theorem 6.3.5]. Invoking our claim (3) above, G_1 is p-nilpotent if $G_1 < G$. This leads to $Q \leq C_G(O_p(G)) \leq O_p(G)$ [9, Theorem 9.3.1], a contradiction. Thus, we have proved that $G = PQ$ is solvable.

(6) The final contradiction.

By step (4), there exists a maximal subgroup M of G such that $G = MN$ and $M \cap N = 1$. Since N is an elementary abelian p-group, $N \leq C_G(N)$ and $C_G(N) \cap M \trianglelefteq G$. By the uniqueness of N, we have $C_G(N) \cap M = 1$ and $N = C_G(N)$. But $N \leq O_p(G) \leq F(G) \leq C_G(N)$, hence $N = O_p(G) = C_G(N)$. Obviously $P = P \cap NM = N(P \cap M)$. Since $P \cap M < P$, we take a maximal subgroup P_1 of P such that $P \cap M \leq P_1$. By our hypotheses, P_1 is weakly ss-permutable in G, then there are a subnormal subgroup T of G and an ss-permutable subgroup $(P_1)_{ss}$ of G contained in P_1 such that $G = P_1T$ and $P_1 \cap T \leq (P_1)_{ss}$. Since $|G : T|$ is a power of p and $T \triangleleft\triangleleft G$, $O^p(G) \leq T$. From the fact that N is the unique minimal normal subgroup of G, we have $N \leq O^p(G) \leq T$. Hence $N \cap P_1 = N \cap (P_1)_{ss}$. By [7, Lemma 2.5], $(P_1)_{ss}G_q = G_q(P_1)_{ss}$ for any Sylow q-subgroup G_q of G, where $q \neq p$. Since $N \cap P_1 = N \cap (P_1)_{ss} = N \cap (P_1)_{ss}G_q$, we have that $N \cap P_1$ is normalized by G_q. Obviously, $N \cap P_1 \trianglelefteq P$. Therefore, $N \cap P_1$ is normal in G. The minimality of N implies that $N \cap P_1 = 1$ or $N \cap P_1 = N$. If $N \cap P_1 = N$, then $N \leq P_1$ and so $P = NP_1 = P_1$, a contradiction. Hence we have $N \cap P_1 = 1$. Since $|N : P_1 \cap N| = |NP_1 : P_1| = |P : P_1| = p$, $P_1 \cap N$ is a maximal subgroup of N.

Therefore $|N| = p$, and so Aut(N) is a cyclic group of order $p-1$. If $q > p$, then NQ is p-nilpotent and therefore $Q \le C_G(N) = N$, a contradiction. On the other hand, if $q < p$, then, since $N = C_G(N)$, we see that $M \cong G/N = N_G(N)/C_G(N)$ is isomorphic to a subgroup of Aut(N) and therefore M, and in particular Q, is cyclic. Since Q is a cyclic group and $q < p$, we know that G is q-nilpotent and therefore P is normal in G. Hence $N_G(P) = G$ is p-nilpotent, a contradiction. □

Corollary 1. *Let p be a prime dividing the order of a group G and H a normal subgroup of G such that G/H is p-nilpotent. If $N_G(P)$ is p-nilpotent and there exists a Sylow p-subgroup P of H such that every maximal subgroup of P is weakly ss-permutable in G, then G is p-nilpotent.*

Proof. It is clear that $N_H(P)$ is p-nilpotent and that every maximal subgroup of P is weakly ss-permutable in H. By Theorem 1, H is p-nilpotent. Now let $H_{p'}$ be the normal Hall p'-subgroup of H. Then $H_{p'}$ is normal in G. If $H_{p'} \neq 1$, then we consider $G/H_{p'}$. It is easy to see that $G/H_{p'}$ satisfies all the hypotheses of our corollary for the normal subgroup $H/H_{p'}$ of $G/H_{p'}$ by Lemma 1. Now by induction, we see that $G/H_{p'}$ is p-nilpotent and so G is p-nilpotent. Hence we may assume $H_{p'} = 1$ and therefore $H = P$ is a p-group. In this case, by our hypotheses, $N_G(P) = G$ is p-nilpotent. □

Theorem 2. *Let p be the smallest prime dividing the order of a group G and P a Sylow p-subgroup of G. If G is A_4-free and every 2-maximal subgroup of P is weakly ss-permutable in G, then G is p-nilpotent.*

Proof. Let P_2 be a 2-maximal subgroup of P. By our hypotheses, P_2 is weakly ss-permutable in G, then there are a subnormal subgroup T of G and an ss-permutable subgroup $(P_2)_{ss}$ of G contained in P_2 such that $G = P_2T$ and $P_2 \cap T \le (P_2)_{ss}$. If P_2 is not ss-permutable in G, then G has a normal subgroup M such that $|G : M| = p$ by Lemma 1(d). It follows that every maximal subgroup of $P \cap M$ is weakly ss-permutable in M by Lemma 1(1). Hence we have that M is p-nilpotent by [3, Theorem 3.1]. It is easy to see that G is p-nilpotent. Now we may assume that every 2-maximal subgroup of P is ss-permutable in G. By [7, Theorem 1.7], we get that G is p-nilpotent too. □

Corollary 2. *Suppose that every 2-maximal subgroup of any Sylow subgroup of a group G is weakly ss-permutable in G. If G is A_4-free, then G is a Sylow tower group of supersolvable type.*

Proof. Let p be the smallest prime dividing $|G|$ and P a Sylow p-subgroup of G. Then every 2-maximal subgroup of P is weakly ss-permutable in G. By Theorem 2, G is p-nilpotent. Let U be the normal p-complement of G. By Lemma 1, U satisfies the hypothesis of the Corollary. It follows by induction that U, and hence G is a Sylow tower group of supersolvable type. □

Corollary 3. *Let p be the smallest prime dividing the order of a group G and G is A_4-free. Assume that H is a normal subgroup of G such that G/H is p-nilpotent. If there exists a Sylow p-subgroup P of H such that every 2-maximal subgroup of P is weakly ss-permutable in G, then G is p-nilpotent.*

Proof. By Lemma 1, every 2-maximal subgroup of P is weakly ss-permutable in H. By Theorem 2, H is p-nilpotent. Now, let $H_{p'}$ be the normal p-complement of H. Then $H_{p'} \trianglelefteq G$. By using the arguments as in the proof of Corollary 1, we may assume that $H_{p'} = 1$ and $H = P$ is a p-group. Since G/H is p-nilpotent, let K/H be the normal p-complement of G/H. By Schur-Zassenhaus's theorem, there exists a Hall p'-subgroup $K_{p'}$ of K such that $K = HK_{p'}$. By Theorem 2, K is p-nilpotent and so $K = H \times K_{p'}$. Hence $K_{p'}$ is a normal p-complement of G. □

Corollary 4. *Let H be a normal subgroup of a group G such that G/H is 2-nilpotent. If there exists a Sylow 2-subgroup P of H such that every 2-maximal subgroup of P is weakly ss-permutable in G and $3 \nmid |G|$, then G is 2-nilpotent.*

Corollary 5. *Let G be a group of odd order and H a normal subgroup of G such that G/H is 3-nilpotent. If there exists a Sylow 3-subgroup P of H such that every 2-maximal subgroup of P is weakly ss-permutable in G, then G is 3-nilpotent.*

Theorem 3. *Let $\mathcal{F}$ be the class of groups with Sylow tower of supersolvable type and G is A_4-free. Then $G \in \mathcal{F}$ if and only if there is a normal subgroup H of G such that $G/H \in \mathcal{F}$ and every 2-maximal subgroup of any Sylow subgroup of H is weakly ss-permutable in G.*

Proof. The necessity is obvious. We only need to prove the sufficiency. Suppose that the assertion is false and let G be a counterexample of minimal order. By Lemma 1, every 2-maximal subgroup of any Sylow subgroup of H is weakly ss-permutable in H. By Corollary 2, H is a Sylow tower group of supersolvable type. Let p be the maximal prime divisor of $|H|$ and let P be a Sylow p-subgroup of H. Then P must be a normal subgroup of G and every 2-maximal subgroup of P is weakly ss-permutable in G. It is easy to see that all 2-maximal subgroups of every Sylow subgroup of H/P are weakly ss-permutable in G/P by Lemma 1 and G/P is A_4-free. By the minimality of G, we have $G/P \in \mathcal{F}$. Let N be a minimal normal subgroup of G contained in P.

(1) $P = N$.

If $N < P$, then $(G/N)/(P/N) \cong G/P \in \mathcal{F}$. We will show that $G/N \in \mathcal{F}$. If $|P/N| \leqslant p^2$, then $G/N \in \mathcal{F}$ by Lemma 5. If $|P/N| > p^2$, then every 2-maximal subgroup of P/N is weakly ss-permutable in G/N by Lemma 1. By the minimality of G, we have $G/N \in \mathcal{F}$. Since $\mathcal{F}$ is a saturated formation, N is the unique minimal normal subgroup of G contained in P and $N \nleq \Phi(G)$. By Lemma 6, it follows that $P = F(P) = N$.

(2) The final contradiction.

Since $N \trianglelefteq G$, we may take a 2-maximal N_2 of N such that $N_2 \trianglelefteq G_p$, where G_p is a Sylow p-subgroup of G. By the hypothesis, N_2 is weakly ss-permutable in G. Then there are a subnormal subgroup T of G and an ss-permutable subgroup $(N_2)_{ss}$ of G contained in N_2 such that $G = N_2T$ and $N_2 \cap T \leq (N_2)_{ss}$. Thus $G = NT$ and $N = N \cap N_2T = N_2(N \cap T)$. This implies that $N \cap T \neq 1$. But since $N \cap T$ is normal in G and N is minimal normal in G, $N \cap T = N$. It follows that $T = G$ and so $N_2 = (N_2)_{ss}$ is ss-permutable in G. By [7, Lemma 2.2], N_2 is s-permutable in G. By Lemma 7, $O^p(G) \leq N_G(N_2)$. Thus $N_2 \trianglelefteq G_pO^p(G) = G$. It follows that $N_2 = 1$ and so $|N| = p^2$. By Lemma 5, $G \in \mathcal{F}$, a contradiction. □

Theorem 4. *Let p be a prime, G be a p-solvable group. If there exists a Sylow p-subgroup P of G such that every maximal subgroup of P is weakly ss-permutable in G, then G is p-supersolvable.*

Proof. Suppose that the theorem is false and let G be a counterexample of minimal order.

(1) G has a unique minimal normal subgroup N such that G/N is p-supersolvable.

Let N be a minimal normal subgroup of G . Since P is the Sylow p-subgroup of G, PN/N is the Sylow p-subgroup of G/N. Let M/N be a maximal subgroup of PN/N, then $M = (M \cap P)N$. Let $P_1 = M \cap P$. Obviously, P_1 is the maximal subgroup of P. Since G is p-solvable, N is an elementary abelian p-group or p'-group. If N is a p'-group, then $M/N = P_1N/N$. If N is a p-group, then $M/N = P_1/N$. By hypothesis, P_1 is weakly ss-permutable in G and so M/N is weakly ss-permutable in G/N by Lemma 1. Hence G/N satisfies all the hypotheses of our theorem. The minimal choice of G implies that G/N is p-supersolvable. Clearly, N is the unique minimal normal subgroup of G as the class of p-supersolvable group is a formation.

(2) $O_{p'}(G) = 1$.

If $O_{p'}(G) \neq 1$, then $G/O_{p'}(G) \cong (G/N)/(O_{p'}(G)/N)$ is p-supersolvable by step (1) and so G is p-supersolvable, a contradiction.

(3) The final contradiction.

Since G is p-solvable, N is an elementary abelian p-group by step (2). If N is contained in all maximal subgroups of G, then $N \leq \Phi(G)$ and so G is p-supersolvable, a contradiction. Hence there exists a maximal subgroup M of G such that $G = NM$ and $N \cap M = 1$. Applying Lemma 8, we have $O_p(G) \cap M \trianglelefteq G$, so that $O_p(G) \cap M = 1$ and $N = O_p(G)$. By using the arguments as in the proof of Theorem 1, we have $|N| = p$ and so G is p-supersolvable. □

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