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ON H_vBE-ALGEBRAS

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Abstract. In this paper, we introduce the notion of H_vBE-algebra and investigate the some properties of it. Also some types of H_vBE-algebras are studied and the relationship between them are stated. We try to show that these notions are independent, by some examples. In addition we show that H_vBE-algebra is an extension of hyper BE-algebra and compute the number of H_vBE-algebras in cases $|H| = 2$ and 3. Furthermore, we study several kinds of homomorphisms on H_vBE-algebras.

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1. Introduction and preliminaries

The theory of hyper structures was introduced by Marty in 1934 during the 8^{th} congress of the Scandinavian Mathematicians[8]. A hyper structure is a non-empty set H, together with a function $\circ : H \times H \longrightarrow P^*(H)$ called hyper operation, where $P^*(H)$ denotes the set of all non-empty subsets of H. Marty introduced hypergroups as a generalization of groups. Some basic definitions and the theorems about hyperstructures can be found in [4, 5]. The concept of H_v structures constitute a generalization of well known algebraic hyper structures where the axioms are replaced by the weak ones. H_v structures were first introduced by Vougiouklis in the forth AHA congress(1990)[14].

H. S. Kim and Y. H. Kim introduced the notation of the BE-algebra as a generalization of dual BCK algebra[7]. Using the notation of upper sets, they gave an equivalent condition of upper sets in BE-algebras and discussed some properties of them. A. Rezaei et al. in [11, 12] show that commutative implicative BE-algebra is equivalent to the commutative self distributive BE-algebra.

Recently R. A. Borzooei et al. introduced the notation of pseudo BE-algebra which is a generalization of BE-algebra[3].They defined the basic concepts of pseudo subalgebras and pseudo filters, and proved that under some conditions, pseudo subalgebra can be a pseudo filter[3].

The goal of this paper is combine the concepts H_v structure with BE-algebra and introducing the H_vBE-algebra as a generalization of hyper BE-algebra, defining the H_v filter and subalgebra in this structure, also it is defined the some types of H_vBE-algebras and described the relationship between them. Finally present the homomorphisms on H_vBE-algebras with considering properties of them.

Definition 1 ([7])**.** Let X be a non-empty set and let "$*$" be a binary operation on X , $1 \in X$. An algebra $(X,*,1)$ of type $(2,0)$ is called a BE-algebra if the following axioms hold:
$(BE1)$ $x*x=1$,
$(BE2)$ $x*1=1$,
$(BE3)$ $1*x=x$,
$(BE4)$ $x*(y*z)=y*(x*z)$, for all $x,y,z \in X$.

We introduce the relation "$\leq$" on X by $x \leq y$ if and only if $x*y=1$.

Proposition 1 ([7])**.** *Let X be a BE-algebra. Then*
(i) $x*(y*x)=1$.
(ii) $y*((y*x)*x)=1$, *for all* $x,y \in X$.

Example 1 ([2]). Let $X=\{1,2,...\}$ and the operation "$*$" be defined as follows:

$$x*y=\begin{cases} 1 & if \quad y \leq x \\ y & otherwise \end{cases}$$

Then $(X,*,1)$ is a BE-algebra.

Definition 2 ([5])**.** Let H be a non-empty set and $\circ : H \times H \longrightarrow P^*(H)$ be a hyper operation. Then $(H,\circ)$ is called an H_v- group if it satisfies the following axioms:
$(H1)$ $x \circ (y \circ z) \bigcap (x \circ y) \circ z \neq \phi$,
$(H2)$ $a \circ H = H \circ a = H$, for all $x,y,z,a \in H$,

where $a \circ H = \bigcup_{h \in H} a \circ h$, $H \circ a = \bigcup_{h \in H} h \circ a$.

Definition 3 ([10])**.** Let H be a non-empty set and $\circ : H \times H \longrightarrow P^*(H)$ be a hyperoperation. Then $(H,\circ,1)$ is called a hyper BE-algebra if satisfies the following axioms:
$(HBE1)$ $x < 1$ *and* $x < x$,
$(HBE2)$ $x \circ (y \circ z) = y \circ (x \circ z)$,
$(HBE3)$ $x \in 1 \circ x$,
$(HBE4)$ $1 < x$ implies $x = 1$, for all $x,y,z \in H$,

where the relation "$<$" is defined by $x < y \iff 1 \in x \circ y$.

2. On H_vBE-algebras

Definition 4. Let H be a non-empty set and $\circ : H \times H \longrightarrow P^*(H)$ be a hyperoperation. Then $(H, \circ, 1)$ is called a H_vBE-algebra if satisfies the following axioms:

(H_vBE1) $x < 1$ *and* $x < x$,

(H_vBE2) $x \circ (y \circ z) \bigcap y \circ (x \circ z) \neq \phi$,

(H_vBE3) $x \in 1 \circ x$,

(H_vBE4) $1 < x$ implies $x = 1$, for all $x, y, z \in H$,

where the relation " $<$ " is defined by $x < y \iff 1 \in x \circ y$.

Also $A < B$ if and only if there exist $a \in A$ and $b \in B$ such that $a < b$.

Example 2. (i)Let $(H, *, 1)$be a BE-algebra. We know that $\circ : H \times H \longrightarrow P^*(H)$ with $x \circ y = \{x * y\}$ is a hyperoperation. Then $(H, \circ, 1)$ is a trivial hyper BE-algebra and a H_vBE-algebra.
(ii) Let $H = \{1,a,b\}$. Define a hyperoperation "$\circ$" as follows:

$\circ$	1	a	b
1	{1}	{a,b}	{b}
a	{1}	{1,a}	{1,b}
b	{1}	{1,a,b}	{1}.

Then $(H, \circ, 1)$ is a H_vBE-algebra.
(iii) Define a hyper operation "$\circ$" on $\mathbb{R}$ as follows:

$$x \circ y = \begin{cases} \{y\} & if \quad x = 1 \\ \mathbb{R} & \quad otherwise \end{cases}$$

Then $(\mathbb{R}, \circ, 1)$ is a H_vBE-algebra.

Proposition 2. *Any hyper BE-algebra is a H_vBE-algebra.*

Proof. It is clear. □

In the following example we show that the converse of Proposition 2 is not true.

Example 3. Define a hyperoperation "$\circ$" on the set $H = \{1,a,b\}$ as follows:

$\circ$	1	a	b
1	{1}	{a}	{b}
a	$\{1, b\}$	{1}	{1,a,b}
b	{1}	{1,b}	{1,b}.

Then $(H,\circ,1)$ is an H_vBE-algebra. And we have that:
$a\circ(b\circ b) = a\circ(\{1,b\}) = \{1,a,b\} \neq \{1,b\} = b\circ(\{1,a,b\}) = b\circ(a\circ b)$.
So $(H,\circ,1)$ does not satisfy $(HBE2)$, and $(H,\circ,1)$ is not a hyper BE-algebra.

Theorem 1. *Let $(H,\circ,1)$ be an H_vBE-algebra. Then*

(i) $A\circ(B\circ C)\bigcap B\circ(A\circ C) \neq \phi$ for every $A,B,C \in P^*(H)$,

(ii) $A < A$,

(iii) $1 < A$ implies $1 \in A$,

(iv) $1 \in x\circ(x\circ x)$,

(v) $x < x\circ x$.

Proof. (i) Let $a \in A, b \in B, c \in C$. Then $a\circ(b\circ c) \subseteq A\circ(B\circ C)$, $b\circ(a\circ c) \subseteq B\circ(A\circ C)$, by (H_vBE2), we have $a\circ(b\circ c)\bigcap b\circ(a\circ c) \neq \phi$. therefore $A\circ(B\circ C)\bigcap B\circ(A\circ C) \neq \phi$.

(ii) Let $a \in A$. Then by (H_vBE1) $A < A$.

(iii) Let $1 < A$. Then there exists an element $a \in A$ such that $1 < a$ by using (H_vBE4) $a = 1$ and so $1 \in A$.

(iv) Let $x \in H$. Then $x < x$, by definition $1 \in x\circ x$ therefore $x\circ 1 \subseteq x\circ(x\circ x)$. Also by (H_vBE1), $x < 1$, so by definition $1 \in x\circ 1$ and then $1 \in x\circ(x\circ x)$.

(v) By (iv) $1 \in x\circ(x\circ x)$. Then there exists $b \in x\circ x$ such that $1 \in x\circ b$ and so $x < b$. □

In the following proposition we compute the number of H_vBE-algebras in two cases.

Proposition 3. *For a set H if*
(i) $|H| = 2$, there exist precisely 2^4 different H_vBE-algebras $(H,\circ,1)$,
(ii) $|H| = 3$, there exist at most $4^7 \times 7^2$ different H_vBE-algebras $(H,\circ,1)$.

Proof. (i) Let $|H| = 2$ and $(H,\circ,1)$ be a H_vBE-algebra. Then $H = \{1,a\}$. Consider the following table:

(I)

$\circ$	1	a
1	A	B
a	C	D

By (H_vBE1) and (H_vBE3), we have $A,C,D \in \{\{1\},\{1,a\}\}$ and $B \in \{\{a\},\{1,a\}\}$.
Thus the cardinality of A,B,C,D is at most 2.
So the number of H_vBE-algebras $(H,\circ,1)$ is at most 2^4.
Now, to determine the number of H_vBE-algebras $(H,\circ,1)$, we must consider the condition of H_v BE-algebra on table (I) for different A,B,C,D, when $A,C,D \in \{\{1\},\{1,a\}\}$ and $B \in \{\{a\},\{1,a\}\}$, that gives 2^4 different tables.One can see that every table introduce a H_vBE-algebra.
In the following we consider two cases of tables:

(1)

$\circ$	1	a
1	$\{1\}$	$\{1,a\}$
a	$\{1,a\}$	$\{1,a\}$

(2)

$\circ$	1	a
1	$\{1,a\}$	$\{1,a\}$
a	$\{1,a\}$	$\{1,a\}$.

In any table,we see that (H_vBE1), (H_vBE3) and (H_vBE4) are obvious.
In tables, we choose x,y,z from $\{1,a\}$ and conclude $x\circ(y\circ z)\bigcap y\circ(x\circ z) \neq \phi$.
For example in (1): $a\circ(1\circ 1) = 1\circ(a\circ 1) = \{1\}$. Similarly in (2): $a\circ(1\circ 1) = \{1,a\} = 1\circ(a\circ 1)$.
(ii): In the following, we compute the number of H_vBE-algebras in three cases:
Case 1: a and b are arbitrary,
Case 2: $a < b$,
Case 3: $b < a$.

Case 1. Let $H = \{1,a,b\}$ and $(H,\circ,1)$ be a H_vBE-algebra. Consider $a\circ a$, we have $1 \in a\circ a$ and $a\circ a \in \{\{1\},\{1,a\},\{1,b\},\{1,a,b\}\}$. Therefore $|a\circ a| \leq 4$.
Similarly, we obtain:

$$max|x\circ y| = \begin{cases} 4 & for \quad x=y=a \\ 7 & for \quad x=a,y=b \\ 4 & for \quad x=a,y=1 \\ 7 & for \quad x=b,y=a \\ 4 & for \quad x=b,y=b \\ 4 & for \quad x=b,y=1 \\ 4 & for \quad x=1,y=a \\ 4 & for \quad x=1,y=b \\ 4 & for \quad x=1,y=1 \end{cases} .$$

So the number of different H_vBE-algebra is at most $4^7 \times 7^2$.

Case 2. If $a < b$, then $1 \in a\circ b$

$$a\circ b \in \{\{1\},\{1,a\},\{1,b\},\{1,a,b\}\}. \ (1)$$

The following array obtained

$$max|x\circ y| = \begin{cases} 4 & for \quad x=y=a \\ 4 & for \quad x=a, y=b \\ 4 & for \quad x=a, y=1 \\ 7 & for \quad x=b, y=a \\ 4 & for \quad x=b, y=b \\ 4 & for \quad x=b, y=1 \\ 4 & for \quad x=1, y=a \\ 4 & for \quad x=1, y=b \\ 4 & for \quad x=1, y=1. \end{cases}$$

Therefore the number of H_vBE-algebras $(H,\circ,1)$ is at most $4^8\times 7$.

Case 3. If $b<a$, in a similar way, we conclude that the number of H_vBE- algebras $(H,\circ,1)$ is at most $4^8\times 7$. □

Notation 1. We see that 1 belongs to any triple combination elements of {1,a,b} in Case 2, for example: $1\in b\circ(a\circ b)\bigcap a\circ(b\circ b)$ because $1\in a\circ b$ then $b\circ 1\subseteq b\circ(a\circ b)$ and $1\in b\circ 1$ therefore $1\in b\circ(a\circ b)$. Also, $1\in b\circ b$ then $1\in a\circ 1\subseteq a\circ(b\circ b)$, therefore $1\in b\circ(a\circ b)\bigcap a\circ(b\circ b)\neq\phi$.

3. SOME TYPES OF H_vBE-ALGEBRAS

In this section, we introduce some types of H_vBE algebras .

Definition 5. A H_vBE-algebra is said to be

(i)a row H_vBE-algebra (briefly, $R-H_vBE$-algebra), if $1\circ x=\{x\}$, for all $x\in H$,

(ii)a column H_vBE-algebra (briefly, $C-H_vBE$-algebra), if $x\circ 1=\{1\}$, for all $x\in H$,

(iii)a diagonal H_vBE-algebra (briefly, $D-H_vBE$-algebra), if $x\circ x=\{1\}$, for all $x\in H$,

(iv)a thin H_vBE-algebra (briefly, $T-H_vBE$-algebra), if it is an $RC-H_vBE$-algebra ($RC-H_v$ means $R-H_v$ and $C-H_v$),

(v)a very thin H_vBE-algebra (briefly,$V-H_vBE$-algebra), if it is an $RCD-H_vBE$-algebra ($RCD-H_v$ means $R-H_v, C-H_v$ and $D-H_v$).

Example 4. (i) Every BE-algebra as $(H,*,1)$ with hyperoperation $x\circ y=\{x*y\}$ is an $RCD-H_vBE$-algebra.

(ii) Let $H=\{1,a\}$ and $H'=\{1,a,b\}$. Define the hyperoperations $\circ_1$ and $\circ_2$ correspond to H and hyperoperations $\circ_3$ and $\circ_4$ correspond to H' as follows:

$\circ_1$	1	a
1	$\{1\}$	$\{a\}$
a	$\{1,a\}$	$\{1\}$

$\circ_2$	1	a
1	$\{1\}$	$\{a\}$
a	$\{1\}$	$\{1,a\}$

$\circ_3$	1	a	b
1	$\{1\}$	$\{a\}$	$\{\mathrm{b}\}$
a	$\{1,b\}$	$\{1\}$	$\{1\}$
b	$\{1,b\}$	$\{1\}$	$\{1\}$

$\circ_4$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{b\}$
b	$\{1\}$	$\{1,a\}$	$\{1\}$

Then $(H,\circ_1,1)$ is a $R-H_vBE$-algebra, $(H,\circ_2,1)$ is an $T-H_vBE$-algebra, $(H',\circ_3,1)$ is a $D-H_vBE$-algebra and $(H',\circ_4,1)$ is a $V-H_vBE$-algebra.

Theorem 2. *Let $(H,\circ,1)$ be a $D-H_vBE$- algebra. Then*
(i) $1\in x\circ a$, for some $a\in 1\circ x$,
(ii) if H be a $C-H_vBE$ algebra, then $1\in y\circ(x\circ y)$, for all $x,y\in H$.

Proof. (i) By Definition 5, $1=1\circ(x\circ x)$ and by (H_vBE2) we have $1\circ(x\circ x)\bigcap x\circ(1\circ x)\neq\phi$ and $1\circ(x\circ x)$ is singelton, then $1\in x\circ(1\circ x)=\bigcup\limits_{a\in 1\circ x} x\circ a$
and $1\in x\circ a$
for some $a\in 1\circ x$.

(ii) By (H_vBE2) and Definition 5, we obtain,
$\phi\neq y\circ(x\circ y)\bigcap x\circ(y\circ y)=y\circ(x\circ y)\bigcap x\circ 1=y\circ(x\circ y)\bigcap\{1\}$
Hence $1\in y\circ(x\circ y)$. □

Proposition 4. *Let $H=\{1,a,b\}$ and $(H,\circ,1)$ be an H_vBE-algebra. Determine the number of non-isomorphic $(H,\circ,1)$ in the following cases.*
(i) $(H,\circ,1)$ is an $R-H_vBE$-algebra,
(ii) $(H,\circ,1)$ is a $C-H_vBE$-algebra,
(iii) $(H,\circ,1)$ is a $D-H_vBE$-algebra,
(iv) $(H,\circ,1)$ is a $T-H_vBE$-algebra,
(v) $(H,\circ,1)$ is a $V-H_vBE$-algebra.

Proof. (i) By Proposition 3 and $1\circ x=\{x\}$, for all $x\in H$, we have the following array:

$$max|x\circ y| = \begin{cases} 4 & for \quad x=y=a \\ 7 & for \quad x=a,y=b \\ 4 & for \quad x=a,y=1 \\ 7 & for \quad x=b,y=a \\ 4 & for \quad x=b,y=b \\ 4 & for \quad x=b,y=1 \\ 1 & for \quad x=1,y=a \\ 1 & for \quad x=1,y=b \\ 1 & for \quad x=1,y=1. \end{cases}$$

Therefore the number of $R-H_vBE$- algebras is at most $4^4\times 7^2$.
(iv) Since $1\circ x=\{x\}$ and $x\circ 1=\{1\}$ for all $x\in H$. We have the following array:

$$max|x\circ y| = \begin{cases} 4 & for \quad x=y=a \\ 7 & for \quad x=a,y=b \\ 1 & for \quad x=a,y=1 \\ 7 & for \quad x=b,y=a \\ 4 & for \quad x=b,y=b \\ 1 & for \quad x=b,y=1 \\ 1 & for \quad x=1,y=a \\ 1 & for \quad x=1,y=b \\ 1 & for \quad x=1,y=1 \end{cases}$$

Hence the number of $T-H_vBE$- algebras is at most $4^2\times 7^2$.
Similarly, for $(ii),(iii)$ and (v) we obtain the numbers $T-H_vBE$- algebras $(4^5\times 7),(4^5\times 7)$ and (7^2) respectively. □

In the next example we explain some relationship among $(R,C,D,T)-H_vBE-$algebras.

Example 5. (i) Every $R-H_vBE$-algebra need not be a $D-H_vBE$-algebra, because, in Example 4, $(H,\circ_2,1)$ is an $R-H_vBE$-algebra but it is not a $D-H_vBE$-algebra.

(ii) Every $RD-H_vBE$-algebra need not be a $C-H_vBE$-algebra, because, in 4 we consider that $(H',\circ_3,1)$ is an $RD-H_v$ BE-algebra, but it is not a $C-H_vBE$-algebra.

(iii) Every $T-H_vBE$- algebra need not be a $D-H_vBE$-algebra, because in 4, we see that $(H,\circ_2,1)$ is a $T-H_vBE$-algebra but it is not a $D-H_vBE$-algebra.

4. WEAK FILTERS IN H_vBE-ALGEBRAS

In [10] it is defined the concept of hyper filters in the hyper BE-algebras. In this section we introduce filters and subalgebras in H_vBE-algebras and state the relationship between them .

Definition 6. Let F be a non-empty subset of a H_vBE-algebra H and $1\in F$. Then F is said to be

(i) a *weak H_v filter* of H if $x\circ y\subseteq F$ and $x\in F$ imply $y\in F$, for all $x,y\in H$.
(ii) a *H_v filter* of H if $x\circ y\approx F(i.e., \phi\neq(x\circ y)\bigcap F)$ and $x\in F$ imply $y\in F$, for all $x,y\in H$.

Example 6. Define hyperoperations "$\circ_1$" and "$\circ_2$" on $H=\{1$,a,b$\}$ as follows:

$\circ_1$	1	a	b
1	{1}	{a,b}	{b}
a	{1}	{1,a}	{1,b}
b	{1}	{1,a,b}	{1}

$\circ_2$	1	a	b
1	{1}	{a,b}	{b}
a	{1}	{1,a,b}	{b}
b	$\{1,b\}$	{1,a,b}	{1,a,b}.

We see that $(H,\circ_1,1)$ is an H_vBE-algebra and $F_1=\{1,a\}$ is a weak H_v filter of H. Also $(H,\circ_2,1)$ is an H_vBE-algebra and $F_2=\{1,a\}$ is an H_v filter of H.

In Example 6, F_1 is not an H_v filter, because $a\circ_1 b\approx F_1$ and $a\in F_1$, but $b\notin F_1$.

Theorem 3. *Every H_v filter is a weak H_v filter.*

Proof. It is straightforward. □

Notation: By Example 6, we can see that the notion of a• *weak H_v filter* and a •*H_v filter* are different in H_vBE-algebras.

Theorem 4. *Let F be a subset of an H_vBE-algebra H and $1\in F$. If $x\circ y<F$ and $x\in F$ implies $y\in F$, for all $x,y\in H$, then F=H.*

Proof. Let x be an arbitrary element of H, by (H_vBE1) and by (H_vBE3),we obtain $x\in 1\circ x$. Since $1\in F$ and $x<1$, we have $1\circ x<F$. By hypothesis, $x\in F$, i.e. , $H\subseteq F$, This prove that $F=H$. □

Definition 7. A subset S of a H_vBE algebra H is said to be a •*subalgebra*, if $x\circ y\subseteq S$, for all $x,y\in S$.

Example 7. In Example 6, {1,b} is a subalgebra of $(H,\circ_1,1)$, but {1,a} is not a subalgebra of $(H,\circ_1,1)$ because $1\circ a\nsubseteq\{1,a\}$.

Theorem 5. *Let H be an H_vBE-algebra and S be a subalgebra of H. Then*
(i) S is a weak H_v filter of H if and only if for all $x\in S$ and
$y\in H\diagdown S, x\circ y\nsubseteq S$.
(ii) S is an H_v filter of H if and only if for all $x\in S$ and $y\in H\diagdown S$,
$x\circ y\not\approx S$.

Proof. (i) Let S be a weak H_v filter of H, $x\in S$, and $y\in H\diagdown S$ and $x\circ y\subseteq S$. Since S is a weak filter and $x\in H$, we have $y\in S$, which is a contradiction.
Conversely, let $x\circ y\nsubseteq S$ where $x\in S$ and $y\in H\diagdown S$. Let $x\circ y\subseteq S$ and $x\in S$. If $y\notin S$, then by assumption , $x\circ y\nsubseteq S$, which is a contradiction.
(ii) Let S be an H_v filter of H, $x\in S$ and $y\in H\diagdown S$, and $x\circ y\approx S$. Since S is an H_v filter and $x\in S$ we have $y\in S$ which is a contradiction.

Conversely, let $x \circ y \not\approx S$ where $x \in S$ and $y \in H \setminus S$. If $x \circ y \approx S$, $x \in S$ and $y \notin S$, then by assumption, $x \circ y \not\approx S$, which is a contradiction. □

In the next examples we show that in general every (weak) H_v filter of H is not a subalgebra and conversely.

Example 8. In Example 6, F_1 and F_2 are both weak H_v filters and H_v filters of H but these are not subalgebras of H.

Example 9. (i) Define a hyperoperation "$\circ$" on $H = \{1, a, b\}$, as follows:

$\circ$	1	a	b
1	$\{1\}$	{a}	{b}
a	$\{1, a, b\}$	$\{1\}$	{a,b}
b	$\{1, a, b\}$	{1,a,b}	$\{1\}$.

We see that $(H, \circ, 1)$ is a $D - H_v - BE$ algebra and $F_1 = \{1, a\}$ is a weak H_v filter. Since $a \circ 1 = \{1, a, b\} \nsubseteq \{1, a\}$, $\{1, a\}$ is not a subalgebra of H.

(iii) Let $H = \{1, a, b\}$ and "$\circ$" be a hyperoperation as follows:

$\circ$	1	a	b
1	$\{1\}$	{a,b}	{b}
a	$\{1\}$	{1,a,b}	{b}
b	$\{1, b\}$	{1,a,b}	{1,a,b} .

We see that $(H, \circ, 1)$ is an H_vBE-algebra and $F_2 = \{1, b\}$ is a subalgebra of H. F_2 is not an H_v filter because $b \circ a = \{1, a, b\}$ and $(b \circ a) \bigcap F_2 \neq \phi, b \in F_2$ but $a \notin F_2$.

5. HOMOMORPHISMS ON H_vBE-ALGEBRAS

Homomorphisms of algebraic hyperstructures are studied by Dresher, Ore, Krasner, Kuntzman, Koskas, Jantosciak, Corsini, Davvaz and many others [1, 5, 6, 9, 13]. In this section, we study several kinds of homomorphisms on Hv BE-algebras.

Definition 8. Let $(H_1, o, 1)$ and $(H_2, *, 1')$ be two H_vBE-algebras.
A map $f : H_1 \to H_2$ is said to be:
(1) a •*homomorphism* or •*inclusion homomorphism* if $f(x \circ y) \subseteq (f(x) * f(y))$ and $f(1) = 1'$, for all $x, y \in H_1$,
(2) a •*good homomorphism* if for all x, y of H_1, we have $f(x \circ y) = f(x) * f(y)$ and $f(1) = 1'$,
(3)an •*isomorphism* if it be an one to one and onto good homomorphism. If f is an •*isomorphism*, then H_1 and H_2 are said to be •*isomorphic*, which is denoted by $H_1 \cong H_2$,

(4) a •*weak homomorphism* if $f(x \circ y) \bigcap (f(x) * f(y)) \neq \phi, f(1) = 1'$, for all $x, y \in H_1$.

Example 10. Let $H_1 = \{1, a, b\}, H_2 = \{1', a', b'\}$. Define hyperoperations "$\circ_1$" and "$\circ_2$" as follows:

$\circ_1$	1	a	b
1	{1}	{a,b}	{b}
a	{1}	{1,a}	{1,b}
b	{1}	{1,a,b}	{1}

$\circ_2$	$1'$	a'	b'
$1'$	$\{1'\}$	$\{a',b'\}$	$\{b'\}$
a'	$\{1',b'\}$	$\{1,a',b'\}$	$\{1',b'\}$
b'	$\{1',b\}$	$\{1',a',b'\}$	$\{1,a',b'\}$.

We see that $(H_1,\circ_1,1)$ and $(H_2,\circ_2,1')$ are H_vBE-algebras.
Let $f:H_1\to H_2$ be defined by $f(1)=1', f(a)=a', f(b)=b'$. Clearly, f is an inclusion homomorphism, but it is not a good homomorphism, because $f(a\circ_1 1)=f(\{1\})=\{1'\}, f(a)\circ_2 f(1)=a'\circ_2 1'=\{1',b'\}$.

Proposition 5. *Let $f:H_1\to H_2$ be a one to one and onto map, $(H_1,\circ,1)$ and $(H_2,*,1')$ are H_vBE-algebras.*
If we have $f(x\circ y)=f(x)*f(y)$, then $f(1)=1'$.

Proof. By (H_vBE4) we know that the element 1 in every H_vBE-algebra is unique.
We must prove that :
(i) $f(1)\in x'*f(1), f(1)\in x'*x'$,
(ii) $x'\in f(1)*x'$,
(iii) $f(1)<x'$ implies $x'=f(1)$, for all $x'\in H_2$.

Since $x'\in H_2$ and f is onto, there exists $x\in H_1$ such that $f(x)=x'$.
By $(HvBE1)$, $x<1$ and hence $1\in x\circ 1$ .Moreover

$$f(1)\in f(x\circ 1)=f(x)*f(1)=x'*f(1).$$

Therefore $f(1)\in x'*f(1)$. The proof of other parts is similar. □

Notation 2. We can see that any *homomorphism* is a *weak homomorphism*, but conversely need not be true.

Example 11. Let $H_1=\{1,a,b\}, H_2=\{1',a',b'\}$. Define hyperoperations "$\circ$" and "$*$" as follows:

$\circ$	1	a	b
1	{1}	{a,b}	{b}
a	{1}	{1,a,b}	{b}
b	$\{1,b\}$	{1,a,b}	{1,a,b}

$*$	$1'$	a'	b'
$1'$	$\{1'\}$	$\{a',b'\}$	$\{b'\}$
a'	$\{1'\}$	$\{1',a'\}$	$\{1',b'\}$
b'	$\{1'\}$	$\{1',a',b'\}$	$\{1'\}$.

We see that $(H_1,\circ,1),(H_2,*,1')$ are H_vBE algebras.
Let $f:H_1\to H_2$ be defined by $f(1)=1', f(a)=a', f(b)=b'$. Then f is a *weak homomorphism*, but it is not an *inclusion homomorphism*, because $f(b\circ 1)=f(\{1,b\})=\{1',b'\}$, Then $f(b)*f(1)=b'*1'=\{1'\}$, therefore $f(b\circ 1)\bigcap(f(b)*f(1))\neq\phi$, But $f(b\circ 1)\nsubseteq f(b)*f(1)$.

6. CONCLUSION

In this present paper, we have introduced the concept of H_vBE-algebras and investigated some of their useful properties.

This work focused on some types of H_vBE-algebras. Also we discuss on H_v filters in this structure and present some fundamental properties that compute number of particular H_vBE-algebras.
In our future work, we will get more results in H_vBE-algebras with applications, and we will define concepts as a quotient, a center in H_vBE-algebras and construct new BE-algebra or H_vBE-algebra.

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