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GENERALIZED WEYL CONFORMAL CURVATURE TENSOR OF GENERALIZED RIEMANNIAN SPACE

NENAD O. VESIĆ

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Abstract. In this paper certain invariants of conformal mappings of the generalized Riemannian space are obtained. Here are the general results. These invariants are the analogues of the Thomas projective parameter and generalized Weyl projective tensor.

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1. Introduction and motivation

A differentiable N-dimensional manifold on which the non-symmetric basic tensor G_{ij} is defined is a generalized Riemannian space. Because of the non-symmetry $G_{ij} \neq G_{ji}$, it is possible to determine the symmetric and antisymmetric part of the metric tensor G_{ij}:

$$g_{ij} = \frac{1}{2}(G_{ij} + G_{ji}) \quad \text{and} \quad F_{ij} = \frac{1}{2}(G_{ij} - G_{ji}). \tag{1.1}$$

Metric g is non-degenerate, i.e. the matrix $\left(g_{ij}\right)_{N\times N}$ is non-singular. The coordinates of contravariant metric tensor g of associated space $\mathbb{R}_N$ are the corresponding inverse matrix elements $\left(g^{ij}\right) = \left(g_{ij}\right)^{-1}$. It also holds that: $g^{i\alpha} g_{j\alpha} = \delta^i_j$.

The use of a non-symmetric basic tensor and a non-symmetric affine connection is motivated by Einstein's and Eisenhart's works [5–7]. In Einstein's Unified Field Theory, the symmetric part g_{ij} of the basic tensor G_{ij} is related to gravitation, and the anti-symmetric part F_{ij} is related to electromagnetism. Some mathematicians have studied and developed the theory of non-symmetric affine connection spaces and the theory of generalized Riemannian spaces, for example, L. P. Eisenhart [7], S. M. Minčić [14, 15], M. Prvanović [17].

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Affine connection coefficients of the space $\mathbb{GR}_N$ are (generalized) Christoffel symbols of the second kind

$$\Gamma^i_{jk} = \frac{1}{2} g^{i\alpha}\big(G_{j\alpha,k} - G_{jk,\alpha} + G_{\alpha k,j}\big), \tag{1.2}$$

where the partial derivative $\partial/\partial x^i$ is marked with a comma. In general, $\Gamma^i_{jk} \neq \Gamma^i_{kj}$ is valid. This coefficient is decomposed in two parts, the symmetric part $\overset{0}{\Gamma}{}^i_{jk}$ and the anti-symmetric part T^i_{jk}, where

$$\overset{0}{\Gamma}{}^i_{jk} = \frac{1}{2}\big(\Gamma^i_{jk} + \Gamma^i_{kj}\big) \quad \text{and} \quad T^i_{jk} = \frac{1}{2}\big(\Gamma^i_{jk} - \Gamma^i_{kj}\big). \tag{1.3}$$

The symmetric part $\overset{0}{\Gamma}{}^i_{jk}$ is the Levi-Civita affine connection of the symmetric metric g and the anti-symmetric part T^i_{jk} is called the torsion tensor.

On the basis of a non-symmetric affine connection, it is possible to consider four types of covariant derivatives with regard to the affine connection of the space $\mathbb{GR}_N$ (see [14, 15]). Afterwards, a family of $\mathbb{GR}_N$ curvature tensors was obtained

$$R^i_{jmn} = \overset{0}{R}{}^i_{jmn} + uT^i_{jm;n} + u'T^i_{jn;m} + vT^\alpha_{jm}T^i_{\alpha n} + v'T^\alpha_{jn}T^i_{\alpha m} + wT^\alpha_{mn}T^i_{\alpha j}, \tag{1.4}$$

where u, u', v, v', w are real constants, a covariant derivative based on an affine connection $\overset{0}{\Gamma}$ is denoted by a semicolon and a curvature tensor

$$\overset{0}{R}{}^i_{jmn} = \overset{0}{\Gamma}{}^i_{jm,n} - \overset{0}{\Gamma}{}^i_{jn,m} + \overset{0}{\Gamma}{}^\alpha_{jm}\overset{0}{\Gamma}{}^i_{\alpha n} - \overset{0}{\Gamma}{}^\alpha_{jn}\overset{0}{\Gamma}{}^i_{\alpha m}. \tag{1.5}$$

Geometrical objects that are invariant with respect to conformal mappings play an important role in the theory of gravity [9, 22]. The aim of our paper is to generalize invariants of conformal mappings from [2, 19]. These mappings generalize the concept of conformal mappings defined by N. S. Sinyukov [18], J. Mikeš [11–13], S. E. Stepanov [20, 21].

2. Conformal mappings of generalized Riemannian space

Suppose

$$f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$$

is a diffeomorphism. We can consider the manifolds $\mathbb{GR}_N$ and $\mathbb{G}\overline{\mathbb{R}}_N$ in *the common system of local coordinates* with respect to this mapping (see Figure 1). Namely, if f maps a point $M \in \mathbb{GR}_N$ to point $\overline{M} \in \mathbb{G}\overline{\mathbb{R}}_N$ and if $(\mathcal{U}, \varphi)$ is a local chart around the point M it will be $\varphi(M) = x = (x^1, \cdots, x^N) \in \mathbb{E}^N$ (Euclidean N-space). In this case, we define (for the coordinate mapping in the $\mathbb{G}\overline{\mathbb{R}}_N$) the mapping $\overline{\varphi} = \varphi \circ f^{-1}$, and then

$$\overline{\varphi}(\overline{M}) = (\varphi \circ f^{-1})(f(M)) = \varphi(M) = x = (x^1, \cdots, x^N) \in \mathbb{E}^N. \tag{2.1}$$

Therefore the points M and $\overline{M} = f(M)$ have the same local coordinates.

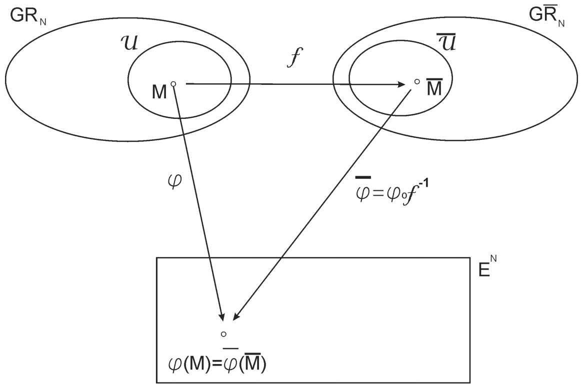

FIGURE 1. Common system of coordinates

Based on Eisenhart's results [7], many authors started the researches on conformal mappings between Riemannian and generalized Riemannian spaces as well as invariants of these mappings (see [1–4, 8, 10–13, 16–21, 23]).

One says that diffeomorphism $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ is **conformal mapping** [19] if the metric tensors G_{ij} and $\overline{G}_{ij}$ of the spaces $\mathbb{GR}_N$ and $\mathbb{G}\overline{\mathbb{R}}_N$ satisfy the condition

$$\overline{G}_{ij} = e^{2\psi} G_{ij}, \tag{2.2}$$

where ψ is a scalar function. The basic equation of conformal mapping f is

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + \psi_j \delta^i_k + \psi_k \delta^i_j - \psi^i g_{jk} + \xi^i_{jk}, \tag{2.3}$$

for $\psi_i = \partial\psi/\partial x^i, \psi^i = g^{i\alpha}\psi_\alpha$, and tensor ξ^i_{jk} anti-symmetric in the indices j and k. After symmetrizing the equation (2.3) by indices j and k and contracting the symmetrized equation by indices i and k, we obtain that is

$$\psi_j = \frac{1}{N}(\overset{0}{\overline{\Gamma}}{}^\alpha_{j\alpha} - \overset{0}{\Gamma}{}^\alpha_{j\alpha}). \tag{2.4}$$

This result proves that it holds

$$\begin{aligned}\overset{0}{\overline{\Gamma}}{}^i_{jk} - \overset{0}{\Gamma}{}^i_{jk} = {} & \frac{1}{N}(\overset{0}{\overline{\Gamma}}{}^\alpha_{j\alpha}\delta^i_k + \overset{0}{\overline{\Gamma}}{}^\alpha_{k\alpha}\delta^i_j - \overset{0}{\overline{\Gamma}}{}^\beta_{\alpha\beta} g^{i\alpha} g_{jk}) \\ & - \frac{1}{N}(\overset{0}{\Gamma}{}^\alpha_{j\alpha}\delta^i_k + \overset{0}{\Gamma}{}^\alpha_{k\alpha}\delta^i_j - \overset{0}{\Gamma}{}^\beta_{\alpha\beta} g^{i\alpha} g_{jk}).\end{aligned} \tag{2.5}$$

Weyl conformal curvature tensor

$$\begin{aligned}\overset{0}{C}{}^i_{jmn} &= \overset{0}{R}{}^i_{jmn} + \frac{1}{N-2}(\delta^i_n \overset{0}{R}_{jm} - \delta^i_m \overset{0}{R}_{jn} + \overset{0}{R}{}^i_n g_{jm} - \overset{0}{R}{}^i_m g_{jn}) \\ &+ \frac{\overset{0}{R}}{(N-1)(N-2)}(\delta^i_m g_{jn} - \delta^i_n g_{jm}),\end{aligned} \tag{2.6}$$

is an invariant of the mapping f.

After anti-symmetrizing the equation (2.3) by indices j and k, we obtain that is $\xi^i_{jk} = \overline{T}^i_{jk} - T^i_{jk}$. If one substitutes this equality in the equation (2.3), the non-symmetric affine connection of the space $\mathbb{GR}_N$ will be reduced to the symmetric affine connection $\overset{0}{\Gamma}$. For this reason, different authors have studied equitorsion conformal mappings (the case of $\xi^i_{jk} = 0$) or conformal mappings of special generalized Riemannian spaces [16, 19, 23]. We will generalize the existing invariants of conformal mappings in this paper.

3. Generalized invariants of conformal mappings

From the equation (2.2), we get

$$\overline{g}_{ij} = e^{2\psi} g_{ij}, \quad \overline{F}_{ij} = e^{2\psi} F_{ij}, \quad \overline{g}^{ij} = e^{-2\psi} g^{ij},$$

which proves that is

$$\overline{g}^{ij}\overline{g}_{mn} = g^{ij} g_{mn} \quad \text{and} \quad \overline{g}^{ij}\overline{F}_{mn} = g^{ij} F_{mn}. \tag{3.1}$$

With regard to the equation (2.5), we obtain that it holds

$$\overset{0}{\overline{\Gamma}}{}^i_{jk} - \overset{0}{\Gamma}{}^i_{jk} = \underset{p}{\overline{\zeta}}{}^i_{jk} - \underset{p}{\zeta}{}^i_{jk}, \tag{3.2}$$

$p = 1, 2$, for

$$\underset{1}{\zeta}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{jk} \quad \text{and} \quad \underset{2}{\zeta}{}^i_{jk} = \frac{1}{N}\big(\overset{0}{\Gamma}{}^\alpha_{j\alpha}\delta^i_k + \overset{0}{\Gamma}{}^\alpha_{k\alpha}\delta^i_j - \overset{0}{\Gamma}{}^\beta_{\alpha\beta} g^{i\alpha} g_{jk}\big), \tag{3.3}$$

and the corresponding $\underset{p}{\overline{\zeta}}{}^i_{jk}$.

From the equations (1.2, 1.3) and the equality $g^{ij}_{;k} = 0$, one obtains that the torsion tensor T^i_{jk} may be expressed as

$$\begin{aligned} T^i_{jk} &= \frac{1}{2} g^{i\alpha}(F_{j\alpha,k} - F_{jk,\alpha} + F_{\alpha k,j}) \\ &= \frac{1}{2}\Big((g^{i\alpha}F_{j\alpha})_{;k} - (g^{i\alpha}F_{k\alpha})_{;j} - (g^{i\alpha}F_{jk})_{;\alpha}\Big). \end{aligned} \tag{3.4}$$

Based on this equation and the second of the equalities (3.1), one establishes that it holds

$$\overline{T}^i_{jk} - T^i_{jk} = \underset{r}{\overline{\tau}}{}^i_{jk} - \underset{r}{\tau}{}^i_{jk} \tag{3.5}$$

$r = (r_1, r_2, r_3, r_4, r_5), r_1, \ldots, r_5 \in \{1,2\}$, for

$$\underset{r}{\tau}{}^i_{jk} = \frac{1}{2} g^{\alpha\beta} \big(\underset{r_1}{\zeta}{}^{\,i}_{\beta k} F_{j\alpha} - \underset{r_2}{\zeta}{}^{\,i}_{\beta j} F_{k\alpha} - \underset{r_3}{\zeta}{}^{\,i}_{\alpha\beta} F_{jk} + \delta^i_\beta \underset{r_4}{\zeta}{}^{\,\gamma}_{j\alpha} F_{\gamma k} - \delta^i_\beta \underset{r_5}{\zeta}{}^{\,\gamma}_{k\alpha} F_{j\gamma} \big)$$

and the corresponding $\underset{r}{\overline{\tau}}{}^i_{jk}$.

Plugging the equations (2.4, 3.5) into the basic equation (2.3), we get

$$\underset{\substack{p\\r}}{\overline{\mathcal{T}}}{}^i_{jk} = \underset{\substack{p\\r}}{\mathcal{T}}{}^i_{jk},$$

for

$$\underset{\substack{p\\r}}{\mathcal{T}}{}^i_{jk} = \Gamma^i_{jk} - \underset{p}{\zeta}{}^i_{jk} - \underset{r}{\tau}{}^i_{jk}, \tag{3.6}$$

the above defined $\underset{p}{\zeta}{}^i_{jk}, \underset{r}{\tau}{}^i_{jk}$ and the corresponding $\underset{\substack{p\\r}}{\overline{\mathcal{T}}}{}^i_{jk}$.

It holds the following lemma:

Lemma 1. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be a conformal mapping. The set of geometrical object given by the equation (3.6) is the set of invariants of the mapping f.* □

Corollary 1. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be an equitorsion conformal mapping. The geometrical object*

$$\overset{0}{\mathcal{T}}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{jk} - \underset{2}{\zeta}{}^i_{jk} \tag{3.7}$$

is an invariant of the mapping f. □

The families R^i_{jmn} and $\overline{R}^i_{jmn}$ of curvature tensors of the spaces $\mathbb{GR}_N$ and $\mathbb{G}\overline{\mathbb{R}}_N$ satisfy the equation

$$\begin{aligned} \overline{R}^i_{jmn} - R^i_{jmn} = {} & \overset{0}{\overline{R}}{}^i_{jmn} - \overset{0}{R}{}^i_{jmn} + u\big(\overline{T}^i_{jm\bar{;}n} - T^i_{jm;n}\big) \\ & + u'\big(\overline{T}^i_{jn\bar{;}m} - T^i_{jn;m}\big) + v\big(\overline{T}^\alpha_{jm}\overline{T}^i_{\alpha n} - T^\alpha_{jm}T^i_{\alpha n}\big) \\ & + v'\big(\overline{T}^\alpha_{jn}\overline{T}^i_{\alpha m} - T^\alpha_{jn}T^i_{\alpha m}\big) + w\big(\overline{T}^\alpha_{mn}\overline{T}^i_{\alpha j} - T^\alpha_{mn}T^i_{\alpha j}\big). \end{aligned} \tag{3.8}$$

With regard to the invariance $\overline{\overset{0}{C}}{}^i_{jmn} = \overset{0}{C}{}^i_{jmn}$, we obtain that is

$$\overline{\overset{0}{R}}{}^i_{jmn} - \overset{0}{R}{}^i_{jmn} = \frac{1}{N-2}\big(\delta^i_n \overset{0}{R}_{jm} - \delta^i_m \overset{0}{R}_{jn} + \overset{0}{R}{}^i_n g_{jm} - \overset{0}{R}{}^i_m g_{jn}\big) - \frac{1}{N-2}\big(\delta^i_n \overline{\overset{0}{R}}_{jm} - \delta^i_m \overline{\overset{0}{R}}_{jn} + \overline{\overset{0}{R}}{}^i_n \overline{g}_{jm} - \overline{\overset{0}{R}}{}^i_m \overline{g}_{jn}\big) + \frac{1}{(N-1)(N-2)}\big(\overset{0}{R}(\delta^i_m g_{jn} - \delta^i_n g_{jm}) - \overline{\overset{0}{R}}(\delta^i_m \overline{g}_{jn} - \delta^i_n \overline{g}_{jm})\big). \tag{3.9}$$

From the equations (3.2, 3.5), we establish that

$$\overline{T}{}^i_{jm\bar{;}n} - T^i_{jm;n} = \overline{\underset{r}{\underset{s}{\sigma}}}{}^i_{jmn} - \underset{r}{\underset{s}{\sigma}}{}^i_{jmn} \tag{3.10}$$

$$\overline{T}{}^\alpha_{jm} \overline{T}{}^i_{\alpha n} - T^\alpha_{jm} T^i_{\alpha n} = \overline{\underset{r^2}{\underset{r^1}{\Theta}}}{}^i_{jmn} - \underset{r^2}{\underset{r^1}{\Theta}}{}^i_{jmn}, \tag{3.11}$$

for $s = (s_1, s_2, s_3), r = (r_1, \dots, r_5), r^u = (r^u_1, \dots, r^u_5), r_v, r^u_v, s_1, s_2, s_3 \in \{1,2\}$,

$$\underset{r}{\underset{s}{\sigma}}{}^i_{jmn} = \underset{r}{\tau}{}^i_{jm;n} + \underset{s_1}{\zeta}{}^i_{\alpha n} (T-\underset{r}{\tau})^\alpha_{jm} - \underset{s_2}{\zeta}{}^\alpha_{jn} (T-\underset{r}{\tau})^i_{\alpha m} - \underset{s_3}{\zeta}{}^\alpha_{mn} (T-\underset{r}{\tau})^i_{j\alpha} \tag{3.12}$$

$$\underset{r^2}{\underset{r^1}{\Theta}}{}^i_{jmn} = T^\alpha_{jm} \underset{r^2}{\tau}{}^i_{\alpha n} + T^i_{\alpha n} \underset{r^1}{\tau}{}^\alpha_{jm} - \underset{r^1}{\tau}{}^\alpha_{jm} \underset{r^2}{\tau}{}^i_{\alpha n}, \tag{3.13}$$

and the corresponding $\overline{\underset{r}{\underset{s}{\sigma}}}{}^i_{jmn}$ and $\overline{\underset{r^2}{\underset{r^1}{\Theta}}}{}^i_{jmn}$.

From equation (1.5) it turns out that:

$$\overset{0}{R}{}^i_{jmn} = R^i_{jmn} - uT^i_{jm;n} - u'T^i_{jn;m} - vT^\alpha_{jm}T^i_{\alpha n} - v'T^\alpha_{jn}T^i_{\alpha m} - wT^\alpha_{mn}T^i_{\alpha j}, \tag{3.14}$$

$$\overset{0}{R}_{ij} = R_{ij} - uT^\alpha_{ij;\alpha} - (v'+w)T^\alpha_{i\beta}T^\beta_{\alpha j}, \tag{3.15}$$

$$\overset{0}{R}{}^i_j = R^i_j - ug^{i\alpha}T^\beta_{\alpha j;\beta} - (v'+w)g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta j}, \tag{3.16}$$

$$\overset{0}{R} = R - (v'+w)T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta}g^{\gamma\delta}, \tag{3.17}$$

for Ricci-tensor $R_{ij} = R^\alpha_{ij\alpha}$, mixed Ricci-curvature tensor $R^i_j = g^{i\alpha}R_{j\alpha}$ and the scalar curvature $R = R^\alpha_\alpha$.

After introducing the equations (3.9–3.17) into the equation (3.8), one obtains that

$$\overline{\underset{\rho}{C}}{}^i_{jmn} = \underset{\rho}{C}{}^i_{jmn},$$

for

$$
\begin{aligned}
\underset{\rho}{C}{}^{i}_{jmn} &= R^{i}_{jmn} + \frac{1}{N-2}(\delta^{i}_{m}R_{jn} - \delta^{i}_{n}R_{jm} + R^{i}_{m}g_{jn} - R^{i}_{n}g_{jm}) \\
&+ \frac{R}{(N-1)(N-2)}(\delta^{i}_{m}g_{jn} - \delta^{i}_{n}g_{jm}) \\
&- \frac{u}{N-2}\big(\delta^{i}_{m}T^{\alpha}_{jn;\alpha} - \delta^{i}_{n}T^{\alpha}_{jm;\alpha} + g^{i\alpha}T^{\beta}_{\alpha m;\beta}g_{jn} - g^{i\alpha}T^{\beta}_{\alpha n;\beta}g_{jm}\big) \\
&- \frac{v'+w}{N-2}\big(\delta^{i}_{m}T^{\alpha}_{j\beta}T^{\beta}_{\alpha n} - \delta^{i}_{n}T^{\alpha}_{j\beta}T^{\beta}_{\alpha m} + g^{i\alpha}T^{\beta}_{\alpha\gamma}(T^{\gamma}_{\beta m}g_{jn} - T^{\gamma}_{\beta n}g_{jm})\big) \\
&- \frac{v'+w}{(N-1)(N-2)}T^{\alpha}_{\gamma\beta}T^{\beta}_{\alpha\delta}g^{\gamma\delta}(\delta^{i}_{m}g_{jn} - \delta^{i}_{n}g_{jm}) \\
&- \underset{\substack{s^1\\r^1}}{\sigma}{}^{i}_{jmn} - u'\underset{\substack{s^2\\r^2}}{\sigma}{}^{i}_{jnm} - v\underset{\substack{r^3\\r^4}}{\Theta}{}^{i}_{jmn} - v'\underset{\substack{r^5\\r^6}}{\Theta}{}^{i}_{jnm} - w\underset{\substack{r^7\\r^8}}{\Theta}{}^{i}_{mnj},
\end{aligned}
\tag{3.18}
$$

$\rho = (s^1, s^2, r^1, \dots, r^8)$, and the corresponding $\underset{\rho}{\overline{C}}{}^{i}_{jmn}$.

The following theorem holds:

Theorem 1. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be a conformal mapping of generalized Riemannian space $\mathbb{GR}_N$. The families $\underset{\rho}{C}{}^{i}_{jmn}$ of geometrical objects given by the equation* (3.18) *are families of invariants of the conformal mapping f.* □

Corollary 2. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be an equitorsion conformal mapping. The family of geometrical objects*

$$
\begin{aligned}
\underset{s}{\widetilde{\mathfrak{C}}}{}^{i}_{jmn} &= R^{i}_{jmn} + \frac{1}{N-2}\big(\delta^{i}_{m}R_{jn} - \delta^{i}_{n}R_{jm} + R^{i}_{m}g_{jn} - R^{i}_{n}g_{jm}\big) \\
&+ \frac{R}{(N-1)(N-2)}\big(\delta^{i}_{m}g_{jn} - \delta^{i}_{n}g_{jm}\big) - u\underset{s^1}{\tilde{\sigma}}{}^{i}_{jmn} - u'\underset{s^2}{\tilde{\sigma}}{}^{i}_{jnm} \\
&- \frac{u}{N-2}\big(\delta^{i}_{m}\underset{s^3}{\tilde{\sigma}}{}^{\alpha}_{jn\alpha} - \delta^{i}_{n}\underset{s^4}{\tilde{\sigma}}{}^{\alpha}_{jm\alpha} + g^{i\alpha}\underset{s^5}{\tilde{\sigma}}{}^{\beta}_{\alpha m\beta}g_{jn} - g^{i\alpha}\underset{s^6}{\tilde{\sigma}}{}^{\beta}_{\alpha n\beta}g_{jm}\big),
\end{aligned}
\tag{3.19}
$$

for

$$
\underset{s^k}{\tilde{\sigma}}{}^{i}_{jmn} = \underset{s^k_1}{\zeta}{}^{i}_{\alpha n}T^{\alpha}_{jm} - \underset{s^k_2}{\zeta}{}^{\alpha}_{jn}T^{i}_{\alpha m} - \underset{s^k_3}{\zeta}{}^{\alpha}_{mn}T^{i}_{j\alpha},
$$

$k = 1, \dots, 6$, *is the family of invariants of the mapping f.* □

4. Conclusion

Weyl conformal curvature tensor was generalized in this paper. We also obtained invariants of conformal mappings analogous to the Thomas projective parameter.

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Author's address

Nenad O. Vesić
University of Nish, Faculty of Science and Mathematics, Višegradska 33, 18000 Niš, Serbia
E-mail address: n.o.vesic@outlook.com