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Decompostion of $(1,2)^*$-continuity and $(1,2)^*$-α-continuity

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DECOMPOSITION OF $(1,2)*$-CONTINUITY AND $(1,2)*$-α-CONTINUITY

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Abstract. In this paper, we introduce new types of sets called $(1,2)*$-D(α,p) sets, $(1,2)*$-D(α,s) sets, $(1,2)*$-D(c,α) sets, $(1,2)*$-D(c,s) sets and $(1,2)*$-D(c,p) sets and new classes of mappings called $(1,2)*$-D(α,p) continuous, $(1,2)*$-D(α,s) continuous, $(1,2)*$-D(c,α) continuous, $(1,2)*$-D(c,s) continuous and $(1,2)*$-D(c,p) continuous mappings. We obtain several characterizations of these classes, study their bitopological properties, and investigate their relation with other bitopological sets and mappings.

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Keywords: $(1,2)*$-D(α,p) set, $(1,2)*$-D(α,s) set, $(1,2)*$-D(c,α) set, $(1,2)*$-D(c,s) set, $(1,2)*$-D(c,p) set, $(1,2)*$-D(α,p) continuous mapping, $(1,2)*$-D(α,s) continuous mapping, $(1,2)*$-D(c,α) continuous mapping, $(1,2)*$-D(c,s) continuous mapping, $(1,2)*$-D(c,p) continuous mapping

1. INTRODUCTION

Njastad [7] intiated the study of the notion of a nearly open set in a topological space. Following it, a number of research papers were written by Tong [11, 12], Hatir and Noiri [3–5], Przemski [8], Dontchev and Przemski [1] and Ganster [2] where decompositions of continuity in topological spaces are considered. Here, following these lines, we study the "decomposition of $(1,2)*$- continuity and $(1,2)*$-α-continuity" and deal with new types of sets such as $(1,2)*$-D(α,p) sets, $(1,2)*$-D(α,s) sets, $(1,2)*$-D(c,α) sets, $(1,2)*$-D(c,s) sets and $(1,2)*$-D(c,p) sets, as well as with new classes of mappings such as $(1,2)*$-D(α,p) continuous, $(1,2)*$-D(α,s) continuous, $(1,2)*$-D(c,α) continuous, $(1,2)*$-D(c,s) continuous and $(1,2)*$-D(c,p) continuous mappins. In this paper, we obtain some important results in bitopological spaces. In most of the occasions, our ideas are illustrated and substantiated by suitable examples.

2. PRELIMINARIES

Let (X,τ_1,τ_2) and (Y,σ_1,σ_2), briefly X and Y, be bitopological spaces.

Definition 1 ([6]). Let S be a subset of X. Then S is called $\tau_{1,2}$-open if $S = A \cup B$, where $A \in \tau_1$ and $B \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Definition 2 ([6]). Let S be a subset of X. Then:

(i) the $\tau_1\tau_2$-closure of S, denoted by $\tau_1\tau_2$-$\mathrm{Cl}\,S$, is defined by

$$\cap\big\{F \mid S \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\big\};$$

(ii) the $\tau_1\tau_2$-interior of S, denoted by $\tau_1\tau_2$-$\mathrm{Int}\,S$, is defined by

$$\cup\big\{F \mid F \subseteq S \text{ and } F \text{ is } \tau_{1,2}\text{-open}\big\}.$$

Note 1 ([6]). Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 3. A subset S of X is called:

(i) $(1,2)*$-α-open [6] if $S \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(S)))$;
(ii) $(1,2)*$-semi-open [6] if $S \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(S))$;
(iii) $(1,2)*$-preopen [6] if $S \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(S))$;
(iv) $(1,2)*$-α-closed [10] if $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(S))) \subseteq S$;
(v) $(1,2)*$-semi-closed [10] if $\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(S)) \subseteq S$;
(vi) $(1,2)*$-preclosed [9] if $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-}\int(S)) \subseteq S$.

The family of $(1,2)*$-α-open sets (resp. $(1,2)*$-semi-open sets, $(1,2)*$-preopen sets) of X is denoted by $(1,2)*$-$\alpha O(X)$ (resp. $(1,2)*$-$SO(X)$, $(1,2)*$-$PO(X)$).

The complement of $(1,2)*$-α-open (resp. $(1,2)*$-semi-open, $(1,2)*$-preopen) set is $(1,2)*$-α-closed (resp. $(1,2)*$-semi-closed, $(1,2)*$-preclosed).

The intersection of all $(1,2)*$-α-closed (resp. $(1,2)*$-semi-closed, $(1,2)*$-preclosed) sets containing A is called the $(1,2)*$-α-closure (resp. $(1,2)*$-semi-closure, $(1,2)*$-preclosure) of A and is denoted by $(1,2)*$-$\alpha\,\mathrm{Cl}(A)$ [resp. $(1,2)*$-$\mathrm{s\,Cl}(A)$, $(1,2)*$-$\mathrm{p\,Cl}(A)$).

The union of all $(1,2)*$-α-open (resp. $(1,2)*$-semi-open, $(1,2)*$-preopen) sets contained in A is called the $(1,2)*$-α-interior (resp. $(1,2)*$-semi-interior, $(1,2)*$-preinterior) of A and is denoted by $(1,2)*$-$\alpha\,\mathrm{Int}(A)$ (resp. $(1,2)*$-$\mathrm{s\,Int}(A)$, $(1,2)*$-$\mathrm{p\,Int}(A)$).

Result 1 ([9]). *Let S be a subset of X. Then:*

(i) $(1,2)*\text{-s}\,\mathrm{Cl}(S) = S \cup \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(S))$;
(ii) $(1,2)*\text{-s}\,\mathrm{Int}(S) = S \cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-}\int(S))$.

3. A NEW TYPE OF SETS

Proposition 1. *For any subset A of a bitopological space X, the followings hold:*

(i) $(1,2)*\text{-}\alpha\,\mathrm{Cl}(A) = A \cup \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$;
(ii) $(1,2)*\text{-}\alpha\,\mathrm{Int}(A) = A \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$;
(iii) $(1,2)*\text{-p}\,\mathrm{Cl}(A) = A \cup \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$;
(iv) $(1,2)*\text{-p}\,\mathrm{Int}(A) = A \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$.

Proof. (i) Since $(1,2)*$-$\alpha\,\mathrm{Cl}(A)$ is $(1,2)*$-α-closed set, $A \subseteq (1,2)*$-$\alpha\,\mathrm{Cl}(A)$. we have $\tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}((1,2)*$-$\alpha\,\mathrm{Cl}(A)))) \subseteq (1,2)*$-$\alpha\,\mathrm{Cl}(A)$ and hence $A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(A))) \subseteq (1,2)*$-$\alpha\,\mathrm{Cl}(A)$. On the other hand, we observe that

$$\begin{aligned}
&\tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A \cup \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A))))))\\
&\subseteq \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A)) \cup \tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A)))))\\
&= \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A)) \cup \tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A)))\\
&= \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A)))\\
&\subseteq A \cup \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(\tau_1\tau_2\text{-}\mathrm{Cl}(A))).
\end{aligned}$$

This shows that $A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(A)))$ is $(1,2)*$-α-closed set and so, $(1,2)*$-$\alpha\,\mathrm{Cl}(A) \subseteq A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(A)))$. Thus, $(1,2)*$-$\alpha\,\mathrm{Cl}(A) = A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(A)))$.

(ii) $(1,2)*$-$\alpha\,\mathrm{Cl}(X \setminus A) = (X \setminus A) \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(X \setminus A)))$. Then $X \setminus (1,2)*$-$\alpha\,\mathrm{Cl}(X \setminus A) = (X \setminus (X \setminus A)) \cap (X \setminus \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(X \setminus A))))$. Therefore $(1,2)*$-$\alpha\,\mathrm{Int}(A) = A \cap \tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(A)))$.

(iii) We observe that

$$\begin{aligned}
&\tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(A \cup \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(A))))\\
&\subseteq \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(A)) \cup \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(A))\\
&= \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(A)) \subseteq A \cup \tau_1\tau_2\text{-}\mathrm{Cl}(\tau_1\tau_2\text{-}\mathrm{Int}(A)).
\end{aligned}$$

This shows that the set $A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(A))$ is $(1,2)*$-preclosed and thus $(1,2)*$-$\mathrm{pCl}(A) \subseteq A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(A))$. On the other hand, since $(1,2)*$-$\mathrm{pCl}(A)$ is $(1,2)*$-preclosed, we obtain that $\tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(A)) \subseteq \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}((1,2)*$-$\mathrm{pCl}(A))) \subseteq (1,2)*$-$\mathrm{pCl}(A)$ and hence $A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(A)) \subseteq (1,2)*$-$\mathrm{pCl}(A)$. Thus, $(1,2)*$-$\mathrm{pCl}(A) = A \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(A))$.

(iv) $(1,2)*$-$\mathrm{pCl}(X \setminus A) = (X \setminus A) \cup \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(X \setminus A))$. Then $X \setminus (1,2)*$-$\mathrm{pCl}(X \setminus A) = X \setminus (X \setminus A) \cap (X \setminus \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(X \setminus A)))$. Hence we have $(1,2)*$-$\mathrm{pInt}(A) = A \cap \tau_1\tau_2$-$\mathrm{Int}(\tau_1\tau_2$-$\mathrm{Cl}(A))$. □

We introduce a new type of sets as follows.

Definition 4. For a bitopological space (X, τ_1, τ_2), we define

(i) $(1,2)*$-$\mathrm{D}(\alpha, \mathrm{p}) = \{S \subseteq X : (1,2)*\text{-}\alpha\,\mathrm{Int}(S) = (1,2)*\text{-}\mathrm{pInt}(S)\}$;

(ii) $(1,2)*$-$\mathrm{D}(\alpha, \mathrm{s}) = \{S \subseteq X : (1,2)*\text{-}\alpha\,\mathrm{Int}(S) = (1,2)*\text{-}\mathrm{sInt}(S)\}$.

Proposition 2. *S is a $(1,2)*$-semi-open set if and only if* $\tau_1\tau_2$-$\mathrm{Cl}(S) = \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(S))$.

Proof. Since $S \subseteq \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(S))$, $\tau_1\tau_2$-$\mathrm{Cl}(S) \subseteq \tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(S))$. On the other hand, since $\tau_1\tau_2$-$\mathrm{Int}(S) \subseteq S$, we have $\tau_1\tau_2$-$\mathrm{Cl}(\tau_1\tau_2$-$\mathrm{Int}(S)) \subseteq \tau_1\tau_2$-$\mathrm{Cl}(S)$.

Thus $\tau_1\tau_2$-Cl$(S)=\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(S))$. Conversely, since $S\subseteq\tau_1\tau_2$-Cl$(S)=\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(S))$, S is a $(1,2)\ast$-semi-open. □

4. Comparisons

Remark 1. We have the following diagram:

$$\begin{array}{ccccc} (1,2)\ast\text{-preopen set} & \Leftarrow & (1,2)\ast\text{-}\alpha\text{-open set} & \Rightarrow & (1,2)\ast\text{-semi-open set} \\ \not\Uparrow & & & & \Downarrow \\ (1,2)\ast\text{-D}(\alpha,\text{s})\text{ set} & & & & (1,2)\ast\text{-D}(\alpha,\text{p})\text{ set} \end{array}$$

The following statements enable us to realize that none of the implications in the above diagram is reversible.

Example 1. Let$X=\{a,b,c\}$, $\tau_1=\{\varnothing,X,\{a,b\}\}$ and $\tau_2=\{\varnothing,X,\{b,c\}\}$. Then the sets in $\{\varnothing,X,\{a,b\},\{b,c\}\}$ are $\tau_{1,2}$-open and the sets in $\{\varnothing,X,\{c\},\{a\}\}$ are $\tau_{1,2}$-closed.

Example 2. Let $X=\{a,b,c\}$, $\tau_1=\{\varnothing,X,\{a\}\}$ and $\tau_2=\{\varnothing,X,\{b\}\}$. Then the sets in $\{\varnothing,X,\{a\},\{b\},\{a,b\}\}$ are $\tau_{1,2}$-open and the sets in $\{\varnothing,X,\{c\},\{b,c\},\{a,c\}\}$ are $\tau_{1,2}$-closed.

Example 3. Let $X=\{a,b,c,d\}$, $\tau_1=\{\varnothing,X,\{c,d\}\}$ and $\tau_2=\{\varnothing,X,\{a\}\}$. Then the elements of $\{\varnothing,X,\{a\},\{c,d\},\{a,c,d\}\}$ are $\tau_{1,2}$-open sets and the elements of $\{\varnothing,X,\{b\},\{a,b\},\{b,c,d\}\}$ are $\tau_{1,2}$-closed sets.

Remark 2. $(1,2)\ast$-preopen and $(1,2)\ast$-D(α,p) sets are independent each other. Indeed, in Example 1, clearly $\{b\}$ is $(1,2)\ast$-preopen but it is not $(1,2)\ast$-D(α,p) set. Moreover, $\{a\}$ is $(1,2)\ast$-D(α,p) set but it is not $(1,2)\ast$-preopen.

Remark 3. $(1,2)\ast$-semi-open and $(1,2)\ast$-D(α,s) sets are independent each other. Indeed, in Example 2, clearly $\{a,c\}$ is $(1,2)\ast$-semi-open set but it is not $(1,2)\ast$-D(α,s) set. Moreover, $\{c\}$ is $(1,2)\ast$-D(α,s) set but it is not $(1,2)\ast$-semi-open.

Remark 4. $(1,2)\ast$-semi-open and $(1,2)\ast$-preopen sets are independent each other. Indeed, in Example 1, clearly $\{b\}$ is $(1,2)\ast$-preopen set but it is not $(1,2)\ast$-semi-open set. In Example 2, $\{a,c\}$ is $(1,2)\ast$-semi-open set but it is not $(1,2)\ast$-preopen.

Theorem 1. *Let A be a subset of X. Then A is $(1,2)\ast$-α-open set in X if and only if A is $(1,2)\ast$-semi-open and $(1,2)\ast$-preopen in X.*

Proof. Let $A\in(1,2)\ast$-$\alpha O(X)$. By the definition of $(1,2)\ast$-α-open set, we have $A\subset\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ and $A\subset\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$. Therefore $A\in(1,2)\ast$-$PO(X)$ and $A\in(1,2)\ast$-$SO(X)$. Hence $A\in(1,2)\ast$-$PO(X)\cap(1,2)\ast$-$SO(X)$.

Conversely, let $A\in(1,2)\ast$-$SO(X)$. Then, by Proposition 2, $\tau_1\tau_2$-Cl$(A)=\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$. Then $\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))=\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. Let $A\in(1,2)\ast$-$PO(X)$. Then $A\subset\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$. We have $A\subset\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. Thus $A\in(1,2)\ast$-$\alpha O(X)$. □

Example 4. A $(1,2)*$-semi-open set need not be $(1,2)*$-α-open. In Example 2, $\{a,c\}$ is $(1,2)*$-semi-open set but it is not $(1,2)*$-α-open.

Example 5. A $(1,2)*$-preopen set need not be $(1,2)*$-α-open. In Example 1, $\{b\}$ is $(1,2)*$-preopen set but it is not $(1,2)*$-α-open.

Proposition 3. *For a bitopological space* (X,τ_1,τ_2)*, the relation* $(1,2)*\text{-}SO(X) \subseteq (1,2)*\text{-D}(\alpha,\text{p})$ *holds.*

Proof. Let $A \in (1,2)*\text{-}SO(X)$. Then, by virtue of Proposition 2, $\tau_1\tau_2\text{-Cl}(A) = \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$. Then $\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)) = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. We have $A \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)) = A \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. Hence, $(1,2)*\text{-p}\,\text{Int}(A) = (1,2)*\text{-}\alpha\,\text{Int}(A)$ by Proposition 1. Therefore A is a $(1,2)*\text{-D}(\alpha,\text{p})$ set. □

Example 6. A $(1,2)*\text{-D}(\alpha,\text{p})$ set need not be $(1,2)*$-semi-open. In Example 2, $\{a\}$ is $(1,2)*\text{-D}(\alpha,\text{p})$ set but it is not $(1,2)*$-semi-open.

Remark 5. $(1,2)*\text{-D}(\alpha,\text{s})$ and $(1,2)*$-preopen are independent each other. Indeed, in Example 3, clearly $\{a,b,c\}$ is $(1,2)*$-preopen but it is not $(1,2)*\text{-D}(\alpha,\text{s})$. Moreover, $\{b\}$ is $(1,2)*\text{-D}(\alpha,\text{s})$ but it is not $(1,2)*$-preopen.

Theorem 2. *For a bitopological space* (X,τ_1,τ_2)*, the equality* $(1,2)*\text{-}\alpha O(X) = (1,2)*\text{-}PO(X) \cap (1,2)*\text{-D}(\alpha,\text{p})$ *holds.*

Proof. Let $A \in (1,2)*\text{-}PO(X) \cap (1,2)*\text{-D}(\alpha,\text{p})$. Then $A = (1,2)*\text{-p}\,\text{Int}(A)$ and $(1,2)*\text{-}\alpha\,\text{Int}(A) = (1,2)*\text{-p}\,\text{Int}(A)$. Hence $A = (1,2)*\text{-}\alpha\,\text{Int}(A)$. Therefore $A \in (1,2)*\text{-}\alpha O(X)$.

Conversely, let $A \in (1,2)*\text{-}\alpha O(X)$. Then $A = (1,2)*\text{-}\alpha\,\text{Int}(A) \subseteq (1,2)*\text{-p}\,\text{Int}(A)$. But $(1,2)*\text{-p}\,\text{Int}(A) \subseteq A$. Therefore $A \in (1,2)*\text{-}PO(X)$ and thus we have $A = (1,2)*\text{-}\alpha\,\text{Int}(A) = (1,2)*\text{-p}\,\text{Int}(A)$. Hence A is a $(1,2)*\text{-D}(\alpha,\text{p})$ set. □

Theorem 3. *For a bitopological space* (X,τ_1,τ_2)*, the equality* $(1,2)*\text{-}\alpha O(X) = (1,2)*\text{-}SO(X) \cap (1,2)*\text{-D}(\alpha,\text{s})$ *holds.*

Proof. Let $A \in (1,2)*\text{-}\alpha O(X)$. Then $A = (1,2)*\text{-}\alpha\,\text{Int}(A) \subseteq (1,2)*\text{-s}\,\text{Int}(A)$. But $(1,2)*\text{-s}\,\text{Int}(A) \subseteq A$. Thus $A \in (1,2)*\text{-}SO(X)$. Also $A = (1,2)*\text{-}\alpha\,\text{Int}(A) = (1,2)*\text{-s}\,\text{Int}(A)$. Thus $A \in (1,2)*\text{-D}(\alpha,\text{s})$.

Conversely, let $A \in (1,2)*\text{-}SO(X) \cap (1,2)*\text{-D}(\alpha,\text{s})$. Then $A = (1,2)*\text{-s}\,\text{Int}(A)$ and $(1,2)*\text{-}\alpha\,\text{Int}(A) = (1,2)*\text{-s}\,\text{Int}(A)$. Now we have $A = (1,2)*\text{-}\alpha\,\text{Int}(A)$. Therefore $A \in (1,2)*\text{-}\alpha O(X)$. □

5. Another new type of sets

Definition 5. For a bitopological space (X,τ_1,τ_2), we define

(i) $(1,2)*\text{-D}(\text{c},\alpha) = \{S \subseteq X : \tau_1\tau_2\text{-Int}(S) = (1,2)*\text{-}\alpha\,\text{Int}(S)\}$;

(ii) $(1,2)*\text{-D}(\text{c},\text{s}) = \{S \subseteq X : \tau_1\tau_2\text{-Int}(S) = (1,2)*\text{-s}\,\text{Int}(S)\}$;

(iii) $(1,2)*\text{-D(c,p)} = \{S \subseteq X : \tau_1\tau_2\text{-Int}(S) = (1,2)*\text{-pInt}(S)\}$.

Proposition 4. *For a subset A of (X, τ_1, τ_2), the following statements are equivalent:*

(i) *A is a $(1,2)*$-D(c,s) set.*
(ii) $\tau_1\tau_2\text{-Int}(A) = A \cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$.

Proof. (i)⇒(ii): Let A be a $(1,2)*$-D(c,s) set. Then we have $\tau_1\tau_2\text{-Int}(A) = (1,2)*\text{-sInt}(A) = A \cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$ by Result 1.

(ii)⇒(i): Let $\tau_1\tau_2\text{-Int}(A) = A \cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$. Then, by Result 1, $\tau_1\tau_2\text{-Int}(A) = (1,2)*\text{-sInt}(A)$. Therefore, A is a $(1,2)*$-D(c,s) set. □

Proposition 5. *Every singleton $\{x\}$ of (X, τ_1, τ_2) is a $(1,2)*$-D(c,s) set.*

Proof. First suppose that $\{x\}$ is $\tau_{1,2}$-open. Then $\tau_1\tau_2\text{-Int}(\{x\}) = \{x\} = \{x\} \cap \tau_1\tau_2\text{-Cl}(\{x\}) = \{x\} \cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\{x\}))$. Therefore, by Proposition 4, $\{x\}$ is a $(1,2)*$-D(c,s) set.

Now, let $\{x\}$ be not $\tau_{1,2}$-open. Then $\tau_1\tau_2\text{-Int}(\{x\}) = \varnothing = \{x\} \cap \varnothing = \{x\} cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\{x\}))$. Therefore, by Proposition 4, $\{x\}$ is a $(1,2)*$-D(c,s) set. □

Proposition 6. *Every $\tau_{1,2}$-open set is a $(1,2)*$-D(c,s) set.*

Proof. $\tau_1\tau_2\text{-Int}(A) = A = A \cap \tau_1\tau_2\text{-Cl}(A) = A \cap \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)) = (1,2)*\text{-sInt}(A)$ by Result 1. □

Proposition 7. *Every $\tau_{1,2}$-open set is a $(1,2)*$-D(c,α) set.*

Proof. By Proposition 1, $\tau_1\tau_2\text{-Int}(A) = A = \tau_1\tau_2\text{-Int}(A \cap \tau_1\tau_2\text{-Cl}(A)) = A \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)) = A \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))) = (1,2)*\text{-}\alpha\text{Int}(A)$. □

Proposition 8. *For a bitopological space (X, τ_1, τ_2), the following assertions hold:*

(i) $(1,2)*\text{-D(c,s)} \subseteq (1,2)*\text{-D}(c,\alpha)$;
(ii) $(1,2)*\text{-D(c,p)} \subseteq (1,2)*\text{-D}(c,\alpha)$.

Proof. (i) Let $A \in (1,2)*\text{-D(c,s)}$. Then $\tau_1\tau_2\text{-Int}(A) = (1,2)*\text{-sInt}(A) \supseteq (1,2)*\text{-}\alpha\text{Int}(A)$. But $\tau_1\tau_2\text{-Int}(A) \subseteq (1,2)*\text{-}\alpha\text{Int}(A)$ and thus we have $\tau_1\tau_2\text{-Int}(A) = (1,2)*\text{-}\alpha\text{Int}(A)$. Therefore, $A \in (1,2)*\text{-D}(c,\alpha)$.

(ii) Let $A \in (1,2)*\text{-D(c,p)}$. Then we get $\tau_1\tau_2\text{-Int}(A) = (1,2)*\text{-pInt}(A) \supseteq (1,2)*\text{-}\alpha\text{Int}(A)$. But $\tau_1\tau_2\text{-Int}(A) \subseteq (1,2)*\text{-}\alpha\text{Int}(A)$ and thus $\tau_1\tau_2\text{-Int}(A) = (1,2)*\text{-}\alpha\text{Int}(A)$. Therefore $A \in (1,2)*\text{-D}(c,\alpha)$. □

The converses of Propositions 7 and 8 are not true as can be seen from the following example.

Example 7. In Example 1, $\{b\}$ is $(1,2)*$-D(c,α) set but it is not $(1,2)*$-D(c,p). In Example 2, $\{b,c\}$ is $(1,2)*$-D(c,α) set but it is not $(1,2)*$-D(c,s). Finally, in Example 1, $\{b\}$ is $(1,2)*$-D(c,α) set but it is not $\tau_{1,2}$-open.

Remark 6. $(1,2)*$-semi-open and $(1,2)*$-D(c, s) sets are independent each other. Indeed, in Example 2, clearly $\{a,c\}$ is $(1,2)*$-semi-open but it is not $(1,2)*$-D(c, s). Moreover, $\{c\}$ is $(1,2)*$-D(c, s) set but it is not $(1,2)*$-semi-open.

Remark 7. $(1,2)*$-preopen and $(1,2)*$-D(c, p) sets are independent each other. In Example 1, clearly $\{b\}$ is $(1,2)*$-preopen but it is not $(1,2)*$-D(c, p). Moreover, $\{a\}$ is $(1,2)*$-D(c, p) set but it is not $(1,2)*$-preopen.

Remark 8. $(1,2)*$-α-open and $(1,2)*$-D(c,α) sets are independent each other. Indeed, let $X=\{a,b,c\}$ with $\tau_1=\{\varnothing,X,\{c\}\}$ and $\tau_2=\{\varnothing,X,\{b,c\}\}$. Then the sets in $\{\varnothing,X,\{c\},\{b,c\}\}$ are $\tau_{1,2}$-open and the sets in $\{\varnothing,X,\{a\},\{a,b\}\}$ are $\tau_{1,2}$-closed. Clearly $\{a,c\}$ is $(1,2)*$-α-open but it is not $(1,2)*$-D(c,α) set. Moreover, $\{a\}$ is $(1,2)*$-D(c,α) set but it is not $(1,2)*$-α-open.

Remark 9. We have the following diagram from the subsets we defined above and the Remarks given above. Also, None of the implications in the diagram is reversible.

$$\begin{array}{ccccc}
(1,2)*\text{-D(c,s)} & & \not\rightarrow & & (1,2)*\text{-semi-open} \\
\downarrow & & & & \uparrow \\
(1,2)*\text{-D}(c,\alpha) & \leftarrow & \tau_{1,2}\text{-open} & \rightarrow & (1,2)*\text{-}\alpha\text{-open} \\
\uparrow & & & & \downarrow \\
(1,2)*\text{-D(c,p)} & & \not\rightarrow & & (1,2)*\text{-preopen}
\end{array}$$

Theorem 4. *For a subset A of (X,τ_1,τ_2), the following assertions are equivalent:*

(i) *A is $\tau_{1,2}$-open.*
(ii) *A is $(1,2)*$-α-open set and a $(1,2)*$-D(c,α) set.*
(iii) *A is $(1,2)*$-preopen set and a $(1,2)*$-D(c, p) set.*

Proof. The implication (i)⇒(ii) is obvious.
(ii)⇒(iii): Let A be $(1,2)*$-α-open set and a $(1,2)*$-D(c,α) set. Then $A\in(1,2)*$-$PO(X)\cap(1,2)*$-$SO(X)$ and $(1,2)*$-α-Int$(A)=\tau_1\tau_2$-Int(A). By Proposition 1, we have $A\cap\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))=\tau_1\tau_2$-Int(A) and subsequently $A\cap\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))=\tau_1\tau_2$-Int(A). Now by Proposition 1, we have $(1,2)*$-pInt$(A)=\tau_1\tau_2$-Int(A). Therefore A is a $(1,2)*$-D(c, p) set.
(iii)⇒(i): Let A be $(1,2)*$-preopen set and a $(1,2)*$-D(c, p) set. Then by definition $A=(1,2)*$-pInt(A) and $\tau_1\tau_2$-Int$(A)=(1,2)*$-pInt(A). Hence $A=\tau_1\tau_2$-Int(A). Therefore A is $\tau_{1,2}$-open set. □

Remark 10. $(1,2)*$-α-open and $(1,2)*$-D(c, s) sets are independent each other. Indeed, in the example given in Remark 8, clearly $\{a,c\}$ is $(1,2)*$-α-open set but it is not $(1,2)*$-D(c, s) set. Moreover, $\{a\}$ is $(1,2)*$-D(c, s) set but it is not $(1,2)*$-α-open.

Theorem 5. *For a subset A of (X,τ_1,τ_2), the following assertions are equivalent:*

(i) *A is $\tau_{1,2}$-open.*
(ii) *A is $(1,2)*$-α-open set and a $(1,2)*$-D(c, s) set.*

(iii) *A is* $(1,2)*$*-semi-open set and a* $(1,2)*$-D(c,s) *set.*

Proof. The implications (i)⇒(ii) and (ii)⇒(iii) are obvious.
(iii)⇒(i): Let A be $(1,2)*$-semi-open set and a $(1,2)*$-D(c,s) set. Then $\tau_1\tau_2$-Int$(A) = (1,2)*$-sInt$(A) = A \cap \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)) = A \cap \tau_1\tau_2$-Cl$(A)$ by the definition, Proposition 2, and Result 1. Now we have $\tau_1\tau_2$-Int$(A) = A$. Therefore A is $\tau_{1,2}$-open. □

6. Decomposition of $(1,2)*$-continuity and $(1,2)*$-α-continuity

Definition 6 ([6]). A mapping $f : X \to Y$ is called

(i) $(1,2)*$-α-continuous if $f^{-1}(V) \in (1,2)*$-$\alpha O(X)$ for each $\sigma_{1,2}$-open set V of Y;
(ii) $(1,2)*$-semi-continuous if $f^{-1}(V) \in (1,2)*$-$SO(X)$ for each $\sigma_{1,2}$-open set V of Y;
(iii) $(1,2)*$-precontinuous if $f^{-1}(V) \in (1,2)*$-$PO(X)$ for each $\sigma_{1,2}$-open set V of Y.

We introduce new classes of mappings as follows:

Definition 7. A mapping $f : X \to Y$ is called

(i) $(1,2)*$-D$(\alpha$,p) continuous if $f^{-1}(V) \in (1,2)*$-D$(\alpha$,p) for each $\sigma_{1,2}$-open set v of Y;
(ii) $(1,2)*$-D$(\alpha$,s) continuous if $f^{-1}(V) \in (1,2)*$-D$(\alpha$,s) for each $\sigma_{1,2}$-open set v of Y;
(iii) $(1,2)*$-D(c,α) continuous if $f^{-1}(V) \in (1,2)*$-D(c,α) for each $\sigma_{1,2}$-open set v of Y;
(iv) $(1,2)*$-D(c,s) continuous if $f^{-1}(V) \in (1,2)*$-D(c,s) for each $\sigma_{1,2}$-open set v of Y;
(v) $(1,2)*$-D(c,p) continuous if $f^{-1}(V) \in (1,2)*$-D(c,p) for each $\sigma_{1,2}$-open set v of Y.

Theorem 6. *A mapping* $f : X \to Y$ *is* $(1,2)*$*-continuous if and only if*

(i) *it is* $(1,2)*$-α*-continuous and* $(1,2)*$-D(c,α) *continuous.*
(ii) *it is* $(1,2)*$*-precontinuous and* $(1,2)*$-D(c,p) *continuous.*
(iii) *it is* $(1,2)*$-α*-continuous and* $(1,2)*$-D(c,s) *continuous.*
(iv) *it is* $(1,2)*$*-semi-continuous and* $(1,2)*$-D(c,s) *continuous.*

Proof. It is a decomposition of $(1,2)*$-continuity from Theorems 4 and 5. □

Theorem 7. *A mapping* $f : X \to Y$ *is* $(1,2)*$-α*-continuous if and only if*

(i) *it is* $(1,2)*$*-semi-continuous and* $(1,2)*$*-precontinuous.*
(ii) *it is* $(1,2)*$*-precontinuous and* $(1,2)*$-D$(\alpha$,p) *continuous.*
(iii) *it is* $(1,2)*$*-semi-continuous and* $(1,2)*$-D$(\alpha$,s) *continuous.*

Proof. It is a decomposition of $(1,2)*$-α-continuity from Theorems 1, 2 and 3. □

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