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HIGHER-ORDER BOUNDARY VALUE PROBLEMS FOR CARATHÉODORY DIFFERENTIAL INCLUSIONS

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Abstract. In this paper we prove existence results for boundary value problems for higher-order differential inclusion $x^{(n)}(t) \in F(t,x(t))$ with nonlocal boundary conditions, where F is a compact convex L^1-Carathéodory multifunction.

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1. INTRODUCTION

The aim of this paper is to establish the existence of solutions of the higher-order boundary value problems

$$\begin{cases} x^{(n)}(t) \in F(t,x(t)) & \text{a.e. on } [0,1], \\ x^{(i)}(0) = 0, \quad 0 \le i \le n-2, \\ x(\eta) = x(1) \end{cases} \tag{1.1}$$

with $n \ge 2$,

$$\begin{cases} x^{(n)}(t) \in F(t,x(t)) & \text{a.e. on } [0,1], \\ x(0) = x'(\eta), \\ x(1) = x(\tau) \end{cases} \tag{1.2}$$

with $n \ge 2$,

$$\begin{cases} x^{(n)}(t) \in F(t,x(t)) & \text{a.e. on } [0,1], \\ x^{(i)}(0) = x^{(i+1)}(\eta), \quad 2 \le i \le n-2, \\ x(0) = x'(\eta), \\ x(1) = x(\tau) \end{cases} \tag{1.3}$$

with $n \ge 4$, and

$$\begin{cases} x^{(n)}(t) \in F(t,x(t)) & \text{a.e. on } [0,1], \\ x^{(i)}(0) = x^{(i+1)}(\eta), \quad 0 \le i \le n-2 \end{cases} \tag{1.4}$$

with $n \geq 2$, where $F\colon[0,1]\times\mathbb{R}\to 2^{\mathbb{R}}$ is a compact convex L^1-Carathéodory multi-valued map and $(\eta,\tau)\in]0,1[^2$.

Three and four-point boundary value problems for second-order differential inclusions was initiated by Benchohra and Ntouyas [1, 2]. The authors proved the existence of solutions on compact intervals for the problems (1.1) and (1.2) in the particular case $n = 2$. In order to obtain solutions of (1.1) and (1.2), the authors reduce the existence of solutions to the search for fixed points of a suitable multi-valued map on the Banach space $C([0,1],\mathbb{R})$. Indeed, they used the fixed point theorem for condensing maps due to Martelli [5].

In this paper, we extend the results of Benchohra and Ntouyas [1,2] to the nth order boundary value problems and prove the existence of solutions of (1.3) and (1.4). We shall adopt the techniques used by Benchohra and Ntouyas in the previous papers.

2. Preliminaries

Let $(E,\|\cdot\|)$ be a Banach space. We denote by $C([0,1],E)$ the Banach space of continuous functions from $[0,1]$ to E equipped with the norm $\|x\|_\infty := \sup\{\|x(t)\| : t\in[0,1]\}$. A multifunction is said to be measurable if its graph is measurable. For more details on measurability theory, we refer the reader to the book of Castaing and Valadier [3].

Definition 1. A multi-valued map $F\colon[0,1]\times\mathbb{R}\to 2^{\mathbb{R}}$ is said to be an L^1-Carathéodory if

(i) $t\mapsto F(t,x)$ is measurable for all $x\in\mathbb{R}$;
(ii) $x\mapsto F(t,x)$ is upper semi-continuous for almost all $t\in[0,1]$;
(iii) For each $k>0$, there exists $h_k\in L^1([0,1];\mathbb{R}^+)$ such that
$$\|F(t,x)\| := \sup\{\|y\| : y\in F(t,x)\}\leq h_k(t)$$
for all $\|x\|\leq k$ and for almost all $t\in[0,1]$.

Definition 2. Let E be a separable Banach space, X a nonempty subset of E, and $G\colon X\to 2^E$ a multi-valued map. We say that:

(1) G is upper semi-continuous on X if for each $x\in X$ the set $G(x)$ is a nonempty closed subset of E and if, for each open set B of E containing $G(x)$, there exists an open neighborhood V of x such that $G(V)\subset B$.
(2) G has a fixed point if there is $x\in X$ such that $x\in G(x)$.
(3) G is said to be completely continuous if $G(B)$ is relatively compact for every B bounded set of X.
(4) If G is upper semi-continuous, it is said to be condensing map if, for any subset $B\subset X$ with $\alpha(B)\neq 0$, we have
$$\alpha(G(B))<\alpha(B),$$
where α denotes the Kuratowski measure of noncompactness.*

*Note that a completely continuous multivalued map is the easiest example of a condensing map.

It is known that if the multi-valued map G is completely continuous with nonempty compact values, the G is upper semi-continuous if and only if G has a closed graph.

Definition 3. A function $x:[0,1]\to\mathbb{R}$ is said to be solution of (1.1) (resp., (1.2), (1.3), (1.4)) if x is $(n-1)$-times differentiable, $x^{(n-1)}$ is absolutely continuous and x satisfies the conditions of (1.1) (resp., (1.2), (1.3), (1.4)).

Let $\eta\in\mathbb{R}$ and $n\in\mathbb{N}\setminus\{0,1\}$. For the techniques reasoning, we will need, in the sequel, the sequence of functions $(\varphi_p)_{2\le p\le n}$ defined by the following formulas.

For all $t\in[0,1]$, we put

$$\begin{aligned}
\varphi_2(t) &= 1,\\
\varphi_3(t) &= t+\varphi_2(\eta),\\
\varphi_p(t) &= \frac{t^{p-2}}{(p-2)!}+\sum_{k=3}^{p-1}\varphi_{k-1}(\eta)\frac{t^{p-k}}{(p-k)!}+\varphi_{p-1}(\eta).
\end{aligned}$$

Remark 1. The following assertions hold:

(a) For all $t\in[0,1]$ and $k\in\{0,\dots,n-2\}$, $\varphi_n^{(k)}(t)=\varphi_{n-k}(t)$;
(b) For all $k\in\{0,\dots,n-3\}$, $\varphi_{n-k}(0)=\varphi_{n-k-1}(\eta)$;
(c) For all $k\in\{0,\dots,n-2\}$, the function $\varphi_n^{(k)}$ is increasing.

3. Main results

Assume that the following hypotheses hold:

(H_1) $F:[0,1]\times\mathbb{R}\to 2^{\mathbb{R}}$ is an L^1-Carathéodory set-valued map with nonempty compact convex values;
(H_2) There exists a function $m\in L^1([0,1],\mathbb{R}^+)$ such that

$$\|F(t,x)\|\le m(t)$$

for almost all $t\in[0,1]$ and all $x\in\mathbb{R}$.

We shall prove the following main results.

Theorem 1. *If assumptions* (H_1) *and* (H_2) *are satisfied, then problem* (1.1) *has at least one solution on* $[0,1]$.

Theorem 2. *If assumptions* (H_1) *and* (H_2) *are satisfied, then problems* (1.2) *and* (1.3) *have at least one solution on* $[0,1]$.

Theorem 3. *If assumptions* (H_1) *and* (H_2) *are satisfied, then problem* (1.4) *has at least one solution on* $[0,1]$.

4. Proof of the main results

For $y \in C([0,1],\mathbb{R})$, we set

$$S_F(y) := \{g \in L^1([0,1],\mathbb{R}) : g(t) \in F(t,y(t)) \text{ for a. e. } t \in [0,1]\}.$$

In the sequel, we will use the following important lemmas. They will play a crucial role in the proof of the main results.

Lemma 1 ([4])**.** *If* $\dim(E) < \infty$ *and* $F:[0,1]\times E \to 2^E$ *is compact and convex then* $S_F(y) \neq \varnothing$ *for all* $y \in E$.

Lemma 2 ([4])**.** *If* F *satisfies* (H_1) *and* $S_F \neq \varnothing$ *then, for any linear continuous mapping* $\Gamma: L^1([0,1],E) \to C([0,1],E)$*, the convex compact multi-function* $\Gamma \circ S_F: C([0,1],E) \to 2^{C([0,1],E)}$ *has a closed graph.*

Lemma 3 ([5])**.** *Let* $T: E \to 2^E$ *be a convex compact condensing multi-valued mapping. If the set*

$$\Omega := \{y \in E : \lambda y \in T(y) \text{ for some } \lambda > 1\}$$

is bounded, then T *has a fixed point.*

Proof of Theorem 1. By Lemma 1, for $y \in C([0,1],\mathbb{R})$, $S_F(y)$ is nonempty. Let us transform the problem into a fixed point problem. Consider the multi-valued map $T : C([0,1],\mathbb{R}) \to 2^{C([0,1],\mathbb{R})}$ defined as follows: for $y \in C([0,1],\mathbb{R})$, $T(y)$ is the set of all $z \in C([0,1],\mathbb{R})$ such that

$$z(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} g(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!} g(s)ds - \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^1 \frac{(1-s)^{n-1}}{(n-1)!} g(s)ds,$$

where $g \in S_F(y)$. We shall show that T satisfies the assumptions of Lemma 3. The proof will be given in several steps:

Step 1: *$T(y)$ is convex for each $y \in C([0,1],\mathbb{R})$*. Let $h_1, h_2 \in T(y)$, then

$$h_i(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} g_i(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!} g_i(s)ds - \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^1 \frac{(1-s)^{n-1}}{(n-1)!} g_i(s)ds,$$

where $g_i \in S_F(y)$ and $i = 1,2$. Let $0 \le \alpha \le 1$. For all $t \in [0,1]$ we have

$$(\alpha h_1 + (1-\alpha)h_2)(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} (\alpha g_1(s) + (1-\alpha)g_2(s))ds + \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!} (\alpha g_1(s) + (1-\alpha)g_2(s))ds$$

$$-\frac{t^{n-1}}{1-\eta^{n-1}}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}(\alpha g_1(s)+(1-\alpha)g_2(s))ds.$$

The set $S_F(y)$ is convex because F is convex. Hence $\alpha h_1+(1-\alpha)h_2 \in T(y)$.

STEP 2: *T is bounded on bounded sets of $C([0,1],\mathbb{R})$.* Indeed, it is sufficient to show that $T(B_r)$ is bounded for all $r \geq 0$, where $B_r = \{y \in C([0,1],\mathbb{R}) : \|y\|_\infty \leq r\}$. Let $h \in T(B_r)$. For all $t \in [0,1]$ we have

$$h(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!}g(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}}\int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!}g(s)ds - \frac{t^{n-1}}{1-\eta^{n-1}}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}g(s)ds,$$

where $y \in B_r$ and $g \in S_F(y)$. Thus, by (H_2),

$$|h(t)| \leq \int_0^t \frac{(t-s)^{n-1}}{(n-1)!}m(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}}\int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!}m(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}m(s)ds.$$

Then

$$\|h\|_\infty \leq \int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}m(s)ds + \frac{1}{1-\eta^{n-1}}\int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!}m(s)ds + \frac{1}{1-\eta^{n-1}}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}m(s)ds.$$

Hence $T(B_r) \subset B_\delta$, where δ is the right-hand term in the above inequality.

STEP 3: *T sends bounded sets of $C([0,1],\mathbb{R})$ into equicontinuous sets.* Indeed, let $h \in T(B_r)$. For all $t \in [0,1]$, we have

$$h(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!}g(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}}\int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!}g(s)ds - \frac{t^{n-1}}{1-\eta^{n-1}}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}g(s)ds,$$

where $y \in B_r$ and $g \in S_F(y)$. Let $t, s \in [0,1]$ be such that $t < s$. We have

$$|h(s)-h(t)| \leq \int_t^s \frac{(s-\tau)^{n-1}}{(n-1)!}|g(\tau)|d\tau + \int_0^t \frac{(s-\tau)^{n-1}-(t-\tau)^{n-1}}{(n-1)!}|g(\tau)|d\tau + \frac{s^{n-1}-t^{n-1}}{1-\eta^{n-1}}\left(\int_0^\eta \frac{(\eta-\tau)^{n-1}}{(n-1)!}|g(\tau)|d\tau + \int_0^1 \frac{(1-\tau)^{n-1}}{(n-1)!}|g(\tau)|d\tau\right)$$

$$\leq \int_t^s \frac{(1-\tau)^{n-1}}{(n-1)!} m(\tau)d\tau + \int_0^1 \frac{(s-\tau)^{n-1}-(t-\tau)^{n-1}}{(n-1)!} m(\tau)d\tau + \frac{s^{n-1}-t^{n-1}}{1-\eta^{n-1}} \left(\int_0^\eta \frac{(\eta-\tau)^{n-1}}{(n-1)!} m(\tau)d\tau + \int_0^1 \frac{(1-\tau)^{n-1}}{(n-1)!} m(\tau)d\tau \right).$$

The right-hand side of the above inequality converges to 0 as s tends to t. Now, by Steps 1, 2, and 3 combined with the Arzelà–Ascoli theorem, we conclude that T is completely continuous.

STEP 4: *T has a closed graph.* Let $(y_p)_p$ a sequence converging to y and consider a sequence $(h_p)_p$ such that $h_p \in T(y_p)$ and $(h_p)_p$ converges to h. We shall prove that $h \in T(y)$. We have

$$h_p(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} g_p(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!} g_p(s)ds - \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^1 \frac{(1-s)^{n-1}}{(n-1)!} g_p(s)ds,$$

where $g_p \in S_F(y_p)$.

Now, we consider the linear continuous operator $\Gamma\colon L^1([0,1],\mathbb{R}) \to C([0,1],\mathbb{R})$ defined by

$$\Gamma(g)(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} g(s)ds + \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!} g(s)ds - \frac{t^{n-1}}{1-\eta^{n-1}} \int_0^1 \frac{(1-s)^{n-1}}{(n-1)!} g(s)ds.$$

We have $h_p \in \Gamma \circ S_F(y_p)$. From Lemma 2, $\Gamma \circ S_F$ has a closed graph, then $h \in \Gamma \circ S_F(y)$. Thus, there exists a $g \in S_F(y)$ such that

$$h(t) = \Gamma(g)(t), \qquad t \in [0,1],$$

which implies that $h \in T(y)$. Consequently, T is upper semi-continuous.

STEP 5: *The following set is bounded:*

$$\Omega = \left\{ y \in C([0,1],\mathbb{R}) : \lambda y \in T(y) \text{ for some } \lambda > 1 \right\}.$$

Indeed, let $y \in \Omega$. Then

$$y(t) = \lambda^{-1} \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} g(s)ds + \frac{\lambda^{-1}t^{n-1}}{1-\eta^{n-1}} \int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!} g(s)ds - \frac{\lambda^{-1}t^{n-1}}{1-\eta^{n-1}} \int_0^1 \frac{(1-s)^{n-1}}{(n-1)!} g(s)ds.$$

where $g \in S_F(y)$. So, we conclude that

$$\|y\|_\infty \le \lambda^{-1}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}m(s)ds + \frac{\lambda^{-1}}{1-\eta^{n-1}}\int_0^\eta \frac{(\eta-s)^{n-1}}{(n-1)!}m(s)ds + \frac{\lambda^{-1}}{1-\eta^{n-1}}\int_0^1 \frac{(1-s)^{n-1}}{(n-1)!}m(s)ds.$$

This shows that Ω is bounded. Thus, T satisfies all the conditions of Lemma 3. Therefore, T has a fixed point which is a solution of (1.1). □

Proof of Theorem 2. Transform the problem into a fixed point problem. Set for all $t \in [0,1]$

$$\psi_n^g(t) = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!}g(s)ds + \sum_{k=0}^{n-2}\varphi_n^{(k)}(t)\int_0^\eta \frac{(\eta-s)^k}{k!}g(s)ds,$$

where $g \in S_F(y)$. Consider the multi-valued map $T : C([0,1],\mathbb{R}) \to 2^{C([0,1],\mathbb{R})}$ defined as follows: for $y \in C([0,1],\mathbb{R})$, one puts

$$T(y) := \left\{ z \in C([0,1],\mathbb{R}) : z(t) = \psi_n^g(t) + \frac{1+t}{1-\tau}\big(\psi_n^g(\tau) - \psi_n^g(1)\big) \text{ for } t \in [0,1] \right\}.$$

Following the steps of the proof of Theorem 1, we can easily show that T has a fixed point y.

Now we shall show that y is a solution of (1.2) and (1.3). We have

$$y(t) = \psi_n^g(t) + \frac{1+t}{1-\tau}\big(\psi_n^g(\tau) - \psi_n^g(1)\big),$$

where $g \in S_F(y)$. Then $y(1) = y(\tau)$. On the other hand, for $0 \le i \le n-2$ and $t \in [0,1]$, we have

$$\begin{aligned}
[\psi_n^g]^{(i)}(t) &= \int_0^t \frac{(t-s)^{n-i-1}}{(n-i-1)!}g(s)ds + \sum_{k=0}^{n-2}\varphi_n^{(k+i)}(t)\int_0^\eta \frac{(\eta-s)^k}{k!}g(s)ds \\
&= \int_0^t \frac{(t-s)^{n-i-1}}{(n-i-1)!}g(s)ds + \sum_{l=i}^{n+i-2}\varphi_n^{(l)}(t)\int_0^\eta \frac{(\eta-s)^{l-i}}{(l-i)!}g(s)ds \\
&= \int_0^t \frac{(t-s)^{n-i-1}}{(n-i-1)!}g(s)ds + \sum_{l=i}^{n-2}\varphi_n^{(l)}(t)\int_0^\eta \frac{(\eta-s)^{l-i}}{(l-i)!}g(s)ds.
\end{aligned}$$

Then, by Remark 1(a) and (b),

$$\begin{aligned}
[\psi_n^g]^{(i)}(0) &= \int_0^\eta \frac{(\eta-s)^{n-i-2}}{(n-i-2)!}g(s)ds + \sum_{l=i}^{n-3}\varphi_n^{(l)}(0)\int_0^\eta \frac{(\eta-s)^{l-i}}{(l-i)!}g(s)ds \\
&= \int_0^\eta \frac{(\eta-s)^{n-i-2}}{(n-i-2)!}g(s)ds + \sum_{l=i}^{n-3}\varphi_{n-l-1}(\eta)\int_0^\eta \frac{(\eta-s)^{l-i}}{(l-i)!}g(s)ds,
\end{aligned}$$

and, by Remark 1(a),

$$\begin{aligned}[\psi_n^g]^{(i+1)}(\eta) &= \int_0^\eta \frac{(\eta-s)^{n-i-2}}{(n-i-2)!}g(s)ds + \sum_{l=i}^{n-3}\varphi_n^{(l+1)}(\eta)\int_0^\eta \frac{(\eta-s)^{l-i}}{(l-i)!}g(s)ds \\ &= \int_0^\eta \frac{(\eta-s)^{n-i-2}}{(n-i-2)!}g(s)ds + \sum_{l=i}^{n-3}\varphi_{n-l-1}(\eta)\int_0^\eta \frac{(\eta-s)^{l-i}}{(l-i)!}g(s)ds.\end{aligned}$$

Consequently,

$$[\psi_n^g]^{(i+1)}(\eta) = [\psi_n^g]^{(i)}(0), \tag{4.1}$$

which implies that $y(0) = y'(\eta)$ and

$$y^{(i)}(0) = y^{(i+1)}(\eta), \qquad 2 \le i \le n-2, \tag{4.2}$$

whenever $n \ge 4$. Finally, it is clear that

$$y^{(n)}(t) = g(t), \qquad t \in [0,1], \tag{4.3}$$

and, hence,

$$y^{(n)}(t) \in F(t, y(t)), \qquad t \in [0,1], \tag{4.4}$$

as required. □

Proof of Theorem 3. Consider the multi-valued map $T : C([0,1],\mathbb{R}) \to 2^{C([0,1],\mathbb{R})}$ defined as follows: for $y \in C([0,1],\mathbb{R})$, one sets

$$T(y) := \{z \in C([0,1],\mathbb{R}) : z(t) = \psi_n^g(t) \text{ for } t \in [0,1]\}.$$

Following the steps of the proof of Theorem 1, we show that T has a fixed point y. Let us show that y is a solution of (1.4). Indeed, by (4.1), we have (4.2). In view of (4.3), it follows that (4.4) also holds. □

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