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ON THE MONOTONICITY OF q-SCHURER-STANCU TYPE POLYNOMIALS

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Abstract. Some properties of monotonicity and convexity of the q-Schurer -Stancu operators are considered. The paper contains also numerical examples based on Matlab algorithms, which verify these properties.

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1. Preliminaries

In the last decades, the application of q-calculus represents one of the most interesting areas of research in approximation theory. Lupaş [12] introduced in 1987 a q-type of the Bernstein operators and in 1997 another generalization of these operators based on q-integers was introduced by Phillips [16]. Their approximation properties were studied by Videnskiĭ [18], N. Mahmudov [13], T. Acar and A. Aral [1] and O. Dalmanoglu [9, 10]. In time, many authors have been studied new classes of q-generalized operators ([2–4, 6, 7, 17]).

Before proceeding further, we mention some basic definitions and notations from q-calculus. For any fixed real number $q > 0$, the q-integer $[k]_q$, for $k \in \mathbb{N}$ is defined as

$$[k]_q = \begin{cases} \dfrac{1-q^k}{1-q}, & q \neq 1, \\ k, & q = 1. \end{cases}$$

The q-factorial integer and the q-binomial coefficients are :

$$[k]_q! = \begin{cases} [k]_q[k-1]_q \dots [1]_q, & k = 1,2,\dots \\ 1, & k = 0, \end{cases}$$

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[k]_q![n-k]_q!}, \ (n \geq k \geq 0).$$

The q-analoque of $(x-a)_q^n$ is the polynomial

$$(x-a)_q^n = \begin{cases} 1, & \text{if } n=0, \\ (x-a)(x-qa)\dots(x-q^{n-1}a), & \text{if } n\geq 1. \end{cases}$$

Let p be a non-negative integer and let α,β be some real parameters satisfying the conditions $0\leq\alpha\leq\beta$. In 2003, D. Bărbosu [8] introduced for any $f\in C[0,1+p]$ and $x\in[0,1]$ the Schurer-Stancu operators as follows

$$S_{m,p}^{(\alpha,\beta)}(f,q,x) = \sum_{k=0}^{m+p} p_{m,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right),$$

where $p_{m,k}(x) = \left(\begin{array}{c} m+p \\ k \end{array}\right) x^k(1-x)^{m+p-k}$.

Recently, P.N. Agrawal, V. Gupta and A.S. Kumar [5] introduced the class of q-Schurer-Stancu operators. For any $m\in\mathbb{N}$, p a fixed non negative integer number and α,β some real parameters satisfying the conditions $0\leq\alpha\leq\beta$, they constructed the class of generalized q-Schurer-Stancu operators

$$\tilde{S}_{m,p}^{(\alpha,\beta)} : C[0,1+p]\to C[0,1],$$

as follows

$$\tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x) = \sum_{k=0}^{m+p} \tilde{p}_{m,k}(x) f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right),\ x\in[0,1], \tag{1.1}$$

where $\tilde{p}_{m,k}(x) = \left[\begin{array}{c} m+p \\ k \end{array}\right]_q x^k(1-x)_q^{m+p-k}$.

If $\alpha=\beta=0$ the above operators reduce to the Bernstein-Schurer operators introduced by Muraru in [14].

Lemma 1 ([5]). *For the operators defined in (1.1) the following properties hold*

1. $\tilde{S}_{m,p}^{(\alpha,\beta)}(e_0,q,x)=1$,

2. $\tilde{S}_{m,p}^{(\alpha,\beta)}(e_1,q,x) = \dfrac{\alpha}{[m]_q+\beta} + \dfrac{[m+p]_q}{[m]_q+\beta}x$,

3.

$$\tilde{S}_{m,p}^{(\alpha,\beta)}(e_2,q,x) = \frac{\alpha^2}{([m]_q+\beta)^2} + \frac{[m+p]_q^2}{([m]_q+\beta)^2}x^2 + \frac{2\alpha[m+p]_q x}{([m]_q+\beta)^2} + \frac{[m+p]_q x(1-x)}{([m]_q+\beta)^2}.$$

The next result is based on Popoviciu's technique and it is expressed in terms of the first order modulus of continuity.

Theorem 1 ([5])**.** *If* $f \in C[0,1+p]$ *and* $q \in (0,1)$ *then*

$$\left\| \tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x) - f(x) \right\| \leq \frac{5}{4}\omega_f(\delta_m) \tag{1.2}$$

holds, where

$$\delta_m = \frac{1}{[m]_q + \beta}\sqrt{[m+p]_q + 4(q^m[p]_q + \alpha - \beta)^2}.$$

2. MONOTONICITY OF THE q-SCHURER-STANCU OPERATORS

Oruc and Philips [15] showed that for a convex function f on [0,1], the q-Bernstein polynomials are monotonic decreasing. In this section we will prove a similar result for q-Schurer-Stancu operators.

Theorem 2. *Let* f *be a convex and increasing function on* $[0, p+1]$*. Then, for* $0 < q \leq 1$ *and* $\beta \leq \dfrac{[p]_q}{q^p}$*,*

$$\tilde{S}_{m-1,p}^{(\alpha,\beta)}(f,q,x) \geq \tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x), \tag{2.1}$$

for $0 \leq x \leq 1$ *and* $m \geq 2$*.*

Proof. For $0 < q < 1$ we have

$$\prod_{s=0}^{m+p-1}(1-q^s x)^{-1}\left[\tilde{S}_{m-1,p}^{(\alpha,\beta)}(f,q,x) - \tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x)\right]$$

$$= \sum_{k=0}^{m+p-1}\begin{bmatrix} m+p-1 \\ k \end{bmatrix}_q x^k \prod_{s=m+p-k-1}^{m+p-1}(1-q^s x)^{-1} f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)$$

$$- \sum_{k=0}^{m+p}\begin{bmatrix} m+p \\ k \end{bmatrix}_q x^k \prod_{s=m+p-k}^{m+p-1}(1-q^s x)^{-1} f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right).$$

Denote

$$\psi_k(x) = x^k \prod_{s=m+p-k}^{m+p-1}(1-q^s x)^{-1} \tag{2.2}$$

and using the following relation

$$x^k \prod_{s=m+p-k-1}^{m+p-1}(1-q^s x)^{-1} = \psi_k(x) + q^{m+p-k-1}\psi_{k+1}(x)$$

we find

$$\prod_{s=0}^{m+p-1}(1-q^s x)^{-1}\left[\tilde{S}_{m-1,p}^{(\alpha,\beta)}(f,q,x) - \tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x)\right]$$

$$= \sum_{k=0}^{m+p-1} f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)\left[\begin{array}{c} m+p-1 \\ k \end{array}\right]_q \left\{\psi_k(x)+q^{m+p-k-1}\psi_{k+1}(x)\right\}$$

$$-\sum_{k=0}^{m+p} f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right)\left[\begin{array}{c} m+p \\ k \end{array}\right]_q \psi_k(x) = \sum_{k=0}^{m+p-1} f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)\left[\begin{array}{c} m+p-1 \\ k \end{array}\right]_q \psi_k(x)$$

$$+\sum_{k=1}^{m+p} q^{m+p-k} f\left(\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}\right)\left[\begin{array}{c} m+p-1 \\ k-1 \end{array}\right]_q \psi_k(x)$$

$$-\sum_{k=0}^{m+p} f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right)\left[\begin{array}{c} m+p \\ k \end{array}\right]_q \psi_k(x) = \sum_{k=1}^{m+p-1}\left\{ f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)\left[\begin{array}{c} m+p-1 \\ k \end{array}\right]_q\right.$$

$$\left.+q^{m+p-k} f\left(\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}\right)\left[\begin{array}{c} m+p-1 \\ k-1 \end{array}\right]_q - f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right)\left[\begin{array}{c} m+p \\ k \end{array}\right]_q\right\}\psi_k(x)$$

$$+\left\{ f\left(\frac{[m+p-1]_q+\alpha}{[m-1]_q+\beta}\right) - f\left(\frac{[m+p]_q+\alpha}{[m]_q+\beta}\right)\right\}\psi_{m+p}(x)$$

$$+\left\{ f\left(\frac{\alpha}{[m-1]_q+\beta}\right) - f\left(\frac{\alpha}{[m]_q+\beta}\right)\right\}\psi_0(x)$$

$$= \sum_{k=1}^{m+p-1}\left[\begin{array}{c} m+p \\ k \end{array}\right]_q a_k\psi_k(x) + \left\{ f\left(\frac{[m+p-1]_q+\alpha}{[m-1]_q+\beta}\right) - f\left(\frac{[m+p]_q+\alpha}{[m]_q+\beta}\right)\right\}\psi_{m+p}(x)$$

$$+\left\{ f\left(\frac{\alpha}{[m-1]_q+\beta}\right) - f\left(\frac{\alpha}{[m]_q+\beta}\right)\right\}\psi_0(x),$$

where

$$a_k = f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)\frac{[m+p-k]_q}{[m+p]_q} + q^{m+p-k} f\left(\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}\right)\frac{[k]_q}{[m+p]_q}$$
$$- f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right).$$

From (2.2) it is clear that each $\psi_k(x)$ is non-negative on $[0,1]$ for $0 \le q \le 1$ and thus, it suffices to show that each a_k is non-negative.

Since f is convex on $[0, p+1]$, for any t_0, t_1 such that $0 \le t_0 < t_1 \le p+1$ and any λ, $0 < \lambda < 1$, we have

$$f(\lambda t_0 + (1-\lambda)t_1) \le \lambda f(t_0) + (1-\lambda) f(t_1). \tag{2.3}$$

Let $t_0 = \dfrac{[k-1]_q+\alpha}{[m-1]_q+\beta}$, $t_1 = \dfrac{[k]_q+\alpha}{[m-1]_q+\beta}$ and $\lambda = q^{m+p-k}\dfrac{[k]_q}{[m+p]_q}$. Then $0 \le t_0 < t_1 \le p+1$ and $0 < \lambda < 1$ for $1 \le k \le m+p-1$. If we replace them in the relation (2.3), it follows

$$q^{m+p-k}\frac{[k]_q}{[m+p]_q} f\left(\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}\right) + \frac{[m+p-k]_q}{[m+p]_q} f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)$$

$$\geq f\left(q^{m+p-k}\frac{[k]_q}{[m+p]_q}\cdot\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}+\frac{[m+p-k]_q}{[m+p]_q}\cdot\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right).$$

Using the inequality $[k]_q([k-1]_q+\alpha)\geq([k]_q+\alpha)[k-1]_q$ and f increasing function, it follows

$$q^{m+p-k}\frac{[k]_q}{[m+p]_q}f\left(\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}\right)+\frac{[m+p-k]_q}{[m+p]_q}f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right) \tag{2.4}$$

$$\geq f\left(\frac{[k]_q+\alpha}{[m+p]_q}\cdot\frac{q^{m+p-k}[k-1]_q+[m+p-k]_q}{[m-1]_q+\beta}\right)=f\left(\frac{[k]_q+\alpha}{[m+p]_q}\cdot\frac{[m+p-1]_q}{[m-1]_q+\beta}\right).$$

Since f is increasing on $[0,p+1]$ and

$$a_k=f\left(\frac{[k]_q+\alpha}{[m-1]_q+\beta}\right)\frac{[m+p-k]_q}{[m+p]_q}+q^{m+p-k}f\left(\frac{[k-1]_q+\alpha}{[m-1]_q+\beta}\right)\frac{[k]_q}{[m+p]_q}$$

$$-f\left(\frac{[k]_q+\alpha}{[m+p]_q}\cdot\frac{[m+p-1]_q}{[m-1]_q+\beta}\right)+\left\{f\left(\frac{[k]_q+\alpha}{[m+p]_q}\cdot\frac{[m+p-1]_q}{[m-1]_q+\beta}\right)-f\left(\frac{[k]_q+\alpha}{[m]_q+\beta}\right)\right\},$$

from the inequality (2.4) we obtain $a_k\geq 0$, $k=\overline{1,m+p-1}$.

Therefore $\tilde{S}^{(\alpha,\beta)}_{m-1,p}(f,q,x)\geq\tilde{S}^{(\alpha,\beta)}_{m,p}(f,q,x)$.

For $q=1$ and $0\leq x<1$ in a similar way the property (2.1) is verified.

For $q=1$ and $x=1$ we have

$$\tilde{S}^{(\alpha,\beta)}_{m-1,p}(f,1,1)-\tilde{S}^{(\alpha,\beta)}_{m,p}(f,1,1)=f\left(\frac{m+p-1+\alpha}{m-1+\beta}\right)-f\left(\frac{m+p+\alpha}{m+\beta}\right)\geq 0.$$

□

Theorem 3. *If f is convex, then for all $m\geq 1$ and $0<q\leq 1$ it follows*

i) $\tilde{S}^{(\alpha,\beta)}_{m,p}(f,q,x)\geq f(x)$, *for* $x\in[0,1]$, f *increasing on* $[0,1]$ *and* $\beta=\alpha+\varepsilon$, $\varepsilon\in\left[0,q^m[p]_q\right]$;

ii) $\tilde{S}^{(\alpha,\beta)}_{m,p}(f,q,x)\geq f(x)$, *for* $x\in\left[0,\frac{\alpha}{\beta-q^m[p]_q}\right]$, f *increasing on* $[0,1]$ *and* $\beta>\alpha+q^m[p]_q$;

iii) $\tilde{S}^{(\alpha,\beta)}_{m,p}(f,q,x)\geq f(x)$, *for* $x\in\left(\frac{\alpha}{\beta-q^m[p]_q},1\right]$, f *decreasing on* $[0,1]$ *and* $\beta>\alpha+q^m[p]_q$.

Proof. We consider the knotes $x_k=\frac{[k]_q+\alpha}{[m]_q+\beta}$, $0\leq k\leq m+p$. From Lemma 1 it follows

$$\sum_{k=0}^{m+p}\tilde{p}_{m,k}(x)=1,\ \sum_{k=0}^{m+p}\tilde{p}_{m,k}(x)x_k=\frac{\alpha}{[m]_q+\beta}+\frac{[m+p]_q}{[m]_q+\beta}x.$$

Using the convexity of function f we have

$$\tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x) = \sum_{k=0}^{m+p} \tilde{p}_{m,k}(x) f(x_k) \geq f\left(\sum_{k=0}^{m+p} \tilde{p}_{m,k}(x) x_k\right)$$

$$= f\left(\frac{\alpha}{[m]_q+\beta} + \frac{[m+p]_q}{[m]_q+\beta} x\right).$$

The following inequalities hold

a) $\dfrac{\alpha}{[m]_q+\beta} + \dfrac{[m+p]_q}{[m]_q+\beta} x \geq x$ for $\beta = \alpha + \epsilon$, $\epsilon \in [0, q^m[p]]$, $x \in [0,1]$;

b) $\dfrac{\alpha}{[m]_q+\beta} + \dfrac{[m+p]_q}{[m]_q+\beta} x \geq x$ for $\beta > \alpha + q^m[p]_q$, $x \in \left[0, \frac{\alpha}{\beta - q^m[p]_q}\right]$;

c) $\dfrac{\alpha}{[m]_q+\beta} + \dfrac{[m+p]_q}{[m]_q+\beta} x \leq x$ for $\beta > \alpha + q^m[p]_q$, $x \in \left[\frac{\alpha}{\beta - q^m[p]_q}, 1\right]$.

The theorem is proved using the monotony of function f and the inequalities a)-c). □

3. Numerical example

Davis [11] proved that for any convex function f, the classical Bernstein polynomial is convex and the sequence of Bernstein polynomials is monotonic decreasing. Oruc and Philips [15] extend these results for the Bernstein operators in q-calculus for $0 < q \leq 1$. In this section we will verify numerically these properties for the q-Schurer-Stancu operators.

TABLE 1. The q-Schurer-Stancu operators

x	$\tilde{S}_{30,6}^{(3,5)}(f,q,x)$	$\tilde{S}_{90,6}^{(3,5)}(f,q,x)$
0	0.029115231597413	0.026565252311249
0.1	0.089811163243826	0.083383934253406
0.2	0.191687642176340	0.179475155093384
0.3	0.347058215579779	0.326749960898829
0.4	0.570050753689721	0.538887425606676
0.5	0.876786656660820	0.831510780091276
0.6	1.285573442130916	1.222376845885256
0.7	1.817111513724159	1.731579581814080
0.8	2.494715950321931	2.381768592612226
0.9	3.344554197167635	3.198383491142964
1	4.395900582819147	4.209905050209471

In Table 1 are calculated the values of the q-Schurer-Stancu operators $\tilde{S}_{30,6}^{(3,5)}(f,q,x)$ and $\tilde{S}_{90,6}^{(3,5)}(f,q,x)$ for $f(x) = x^3 e^{x+1}$ and $q = 0.9$. Also, in the Figure 1 are given the graphics of these operators.

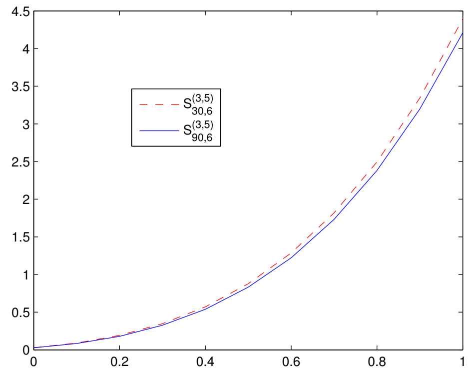

FIGURE 1. The monotonicity of the q-Schurer-Stancu operators

In the next part of this section we will give some numerical examples which verify the inequalities proved in Theorem 3.

Example 1. If $n = 50$, $p = 5$, $\alpha = 3$, $\beta = 3.0211$, $q = 0.9$, $f(x) = x^3 e^{x+1}$, it follows $\tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x) \geq f(x)$ for all $x \in [0,1]$.

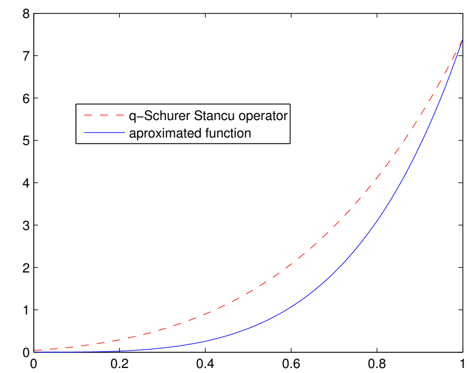

FIGURE 2. The q-Schurer-Stancu operators for increasing function and $\beta = \alpha + \epsilon$

Example 2. If $n=50,\ p=5,\ \alpha=3,\ q=0.9,\ \beta=8.0211,\ f(x)=x^3e^{x+1}$, it follows $\tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x)\geq f(x)$ for all $x\in[0,0.375]$.

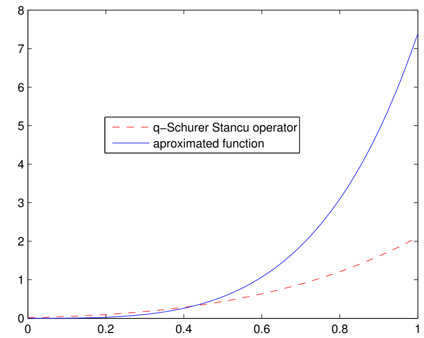

FIGURE 3. The q-Schurer-Stancu operators for increasing function and $\beta>\alpha+q^m[p]_q$

Example 3. If $n=50$, $p=5$, $\alpha=3$, $q=0.9$, $\beta=8.0211$, $f(x)=e^{-x^2}$, it follows $\tilde{S}_{m,p}^{(\alpha,\beta)}(f,q,x)\geq f(x)$ for all $x\in[0.375,1]$.

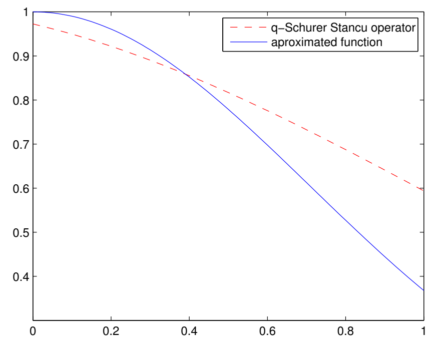

FIGURE 4. The q-Schurer-Stancu operators for decreasing function

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