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MORE ON CONTRA δ-PRECONTINUOUS FUNCTIONS

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Abstract. In [4], Dontchev introduced and investigated a new notion of continuity called contra-continuity. Recently, Jafari and Noiri [8–10] introduced new generalizations of contra-continuity called contra-α-continuity, contra-super-continuity and contra-precontinuity. Recently, Ekici and Noiri [6] have introduced a new class of continuity called contra δ-precontinuity as a generalization of contra-continuity. In this paper, we obtain some more properties of contra δ-precontinuous functions.

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1. INTRODUCTION AND PRELIMINARIES

Recently, Jafari and Noiri have introduced and investigated the notions of contra-precontinuity [10], contra-α-continuity [9] and contra-super-continuity [8] as a continuation of research done by Dontchev [4] and Dontchev and Noiri [5] on the interesting notions of contra-continuity and contra-semi-continuity, respectively. Caldas and Jafari [3] introduced and investigated the notion of contra-β-continuous functions in topological spaces. Raychaudhuri and Mukherjee [15] introduced the notions of δ-preopen sets and δ-almost continuity in topological spaces. The class of δ-preopen sets is larger than one of preopen sets. Recently, by using δ-preopen sets, Ekici and Noiri [6] have introduced the notion of contra δ-precontinuity as a generalization of contra-precontinuity.

In this paper, we obtain the further properties of contra δ-precontinuous functions. Throughout this paper, all spaces (X,τ) and (Y,σ) (or X and Y) are topological spaces. A subset A of X is said to be regular open (resp., regular closed) if $A = \mathrm{Int}(\mathrm{Cl}(A))$ (resp., $A = \mathrm{Cl}(\mathrm{Int}(A))$) where $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure and interior of A. A subset A of a space X is called preopen [12] (resp., semi-open [11], α-open [14], β-open [1]) if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$ (resp., $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$, $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$). The complement of a preopen (resp., semi-open, α-open, β-open) set is said to be preclosed (resp., semi-closed, α-closed, β-closed). The collection of all closed (resp., semi-open, clopen) subsets of X will be denoted by $C(X)$ (resp., $SO(X)$, $CO(X)$). We set $C(X,x) = \{V \in C(X) \mid x \in V\}$ for $x \in X$. We define $CO(X,x)$ in a similar way.

The notion of the δ-closure of A which is denoted by $\delta\operatorname{Cl}(A)$ was introduced by Veličko [19] and is widely investigated in the literature. The δ-closure of A is the set $\{x \in X \mid \operatorname{Int}(\operatorname{Cl}(U)) \cap A \neq \varnothing$ for every open set U containing $x\}$. If $\delta\operatorname{Cl}(A) = A$, then A is said to be δ-closed [19]. The complement of a δ-closed set is said to be δ-open. The union of all δ-open sets contained in A is called the δ-interior of A and is denoted by $\delta\operatorname{Int}(A)$. A subset A of a topological space X is said to be δ-preopen [15] if $A \subset \operatorname{Int}(\delta\operatorname{Cl}(A))$. The complement of a δ-preopen se is said to be δ-preclosed. The intersection (union) of all δ-preclosed (δ-preopen) sets containing (contained in) A in X is called the δ-preclosure (δ-preinterior) of A and is denoted by $\delta\operatorname{Cl}_p(A)$ (resp., $\delta\operatorname{Int}_p(A)$). By $\delta PO(X)$ (resp., $\delta PC(X)$), we denote the collection of all δ-preopen (resp., δ-preclosed) sets of X.

Lemma 1 ([2, 15, 17]). *The following properties holds for the δ-preclosure of a set in a space X:*

(1) *Arbitrary union (intersection) of δ-preopen (δ-preclosed) sets in X is δ-preopen (resp., δ-preclosed).*
(2) *A is δ-preclosed in X iff $A = \delta\operatorname{Cl}_p(A)$.*
(3) *$\delta\operatorname{Cl}_p(A) \subset \delta\operatorname{Cl}_p(B)$ whenever $A \subset B(\subset X)$.*
(4) *$\delta\operatorname{Cl}_p(A)$ is δ-preclosed in X.*
(5) *$\delta\operatorname{Cl}_p(\delta\operatorname{Cl}_p(A)) = \delta\operatorname{Cl}_p(A)$.*
(6) *$\delta\operatorname{Cl}_p(A) = \{x \in X \mid U \cap A \neq \varnothing$ for every δ-preopen set U containing $x\}$.*
(7) *$\delta\operatorname{Cl}_p(A) = A \cup \operatorname{Cl}(\delta\operatorname{Int}(A))$.*
(8) *If A is δ-open, then $\delta\operatorname{Cl}_p(A) = \operatorname{Cl}(A)$.*
(9) *If $Y \subset X$ is δ-open and $U \in \delta PO(Y)$, then $U \in \delta PO(X)$.*
(10) *$U \cap V \in \delta PO(U)$ if U is δ-open and $V \in \delta PO(X)$.*

Definition 1. A function $f: X \to Y$ is said to be contra δ-precontinuous [6] (resp., δ-almost continuous [15]) if $f^{-1}(V)$ is δ-preclosed (resp., δ-preopen) in X for each open set V of Y.

Definition 2. Let A be a subset of a space (X, τ). The set $\cap\{U \in \tau \mid A \subset U\}$ is called the kernel of A [13] and is denoted by $\ker(A)$.

Lemma 2 (Jafari and Noiri [8]). *The following properties hold for subsets A, B of a space X:*

(1) *$x \in \ker(A)$ if and only if $A \cap F \neq \varnothing$ for any $F \in C(X,x)$.*
(2) *$A \subset \ker(A)$ and $A = \ker(A)$ if A is open in X.*
(3) *If $A \subset B$, then $\ker(A) \subset \ker(B)$.*

2. Contra δ-precontinuous functions

Theorem 1. *The following assertions are equivalent for a function $f: X \to Y$:*

(1) f *is contra* δ*-precontinuous.*
(2) *For every closed subset* F *of* Y, $f^{-1}(F) \in \delta PO(X)$.
(3) *For each* $x \in X$ *and each* $F \in C(Y, f(x))$, *there exists* $U \in \delta PO(X,x)$ *such that* $f(U) \subset F$.
(4) $f(\delta \operatorname{Cl}_p(A)) \subset \ker(f(A))$ *for every subset* A *of* X .
(5) $\delta \operatorname{Cl}_p(f^{-1}(B)) \subset f^{-1}(\ker(B))$ *for every subset* B *of* Y.

Proof. The implications (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (4) : Let A be any subset of X. Suppose that $y \notin \ker(f(A))$. Then, by Lemma 2, there exists $F \in C(Y, y)$ such that $f(A) \cap F = \varnothing$. For any $x \in f^{-1}(F)$, by (3) there exists $U_x \in \delta PO(X,x)$ such that $f(U_x) \subset F$. Hence $f(A \cap U_x) \subset f(A) \cap f(U_x) \subset f(A) \cap F = \varnothing$ and $A \cup U_x = \varnothing$. This shows that $x \notin \delta \operatorname{Cl}_p(A)$ for any $x \in f^{-1}(F)$. Therefore, $f^{-1}(F) \cap \delta \operatorname{Cl}_p(A) = \varnothing$ and hence $F \cap f(\delta \operatorname{Cl}_p(A)) = \varnothing$. Thus, $y \notin f(\delta \operatorname{Cl}_p(A))$. Consequently, we obtain $f(\delta \operatorname{Cl}_p(A)) \subset \ker(f(A))$.

(4) $\Rightarrow$ (5) : Let B be any subset of Y. Them, by (4) and Lemma 2, we have $f(\delta \operatorname{Cl}_p(f^{-1}(B))) \subset \ker(f(f^{-1}(B))) \subset \ker(B)$ and therefore $\delta \operatorname{Cl}_p(f^{-1}(B)) \subset f^{-1}(\ker(B))$.

(5) $\Rightarrow$ (1) : Let V be any open set of Y. Then, by virtue of Lemma 2, we have $\delta \operatorname{Cl}_p(f^{-1}(V)) \subset f^{-1}(\ker(V)) = f^{-1}(V)$ and $\delta \operatorname{Cl}_p(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is δ-preclosed in X. □

The following two examples show that δ-almost continuous and contra δ-precontinuous are independent concepts.

Example 1. The identity function on the real line (with the usual topology) is continuous and hence δ-almost continuous but not contra δ-precontinuous, since the preimage of each singleton fails to be δ-preopen.

Example 2. Let $X = \{a,b\}$ be the Sierpinski space endowed with the topology $\tau = \{\varnothing, \{a\}, X\}$. Let $f : X \to X$ be defined by $f(a) = b$ and $f(b) = a$. Since the inverse image of every open set is δ-preclosed, then f is contra δ-precontinuous, but $f^{-1}(\{a\})$ is not δ-preopen in (X,τ). Therefore f is not δ-almost continuous.

Definition 3. A function $f : X \to Y$ is said to be contra-continuous [4] (resp., contra-α-continuous [9], contra-precontinuous [10], contra-semi-continuous [5], contra-β-continuous [3]) if, for each open set V of Y, $f^{-1}(V)$ is closed (resp., α-closed, preclosed, semi-closed, β-closed) in X.

For the functions defined above, we have the following implications:

$$\begin{array}{ccccccc} A & \Longrightarrow & B & \Longrightarrow & C & \Longrightarrow & D \\ & & \Downarrow & & \Downarrow & & \\ & & C & \Longrightarrow & F & & \end{array}$$

The meaning of symbols here is as follows: A = contra-continuity, B = contra-α-continuity, C = contra-precontinuity, D = contra δ-precontinuity, E = contra-semi-continuity, and F = contra-β-continuity.

It should be mentioned that none of these implications is reversible as shown by the examples stated below.

Example 3 (Jafari and Noiri [9]). Let $X = \{a,b,c\}$. Put $\tau = \{\varnothing,\{a\},X\}$ and $\sigma = \{\varnothing,\{b\},\{c\},\{b,c\},X\}$. Then the identity function $f:(X,\tau) \to (X,\sigma)$ is contra-α-continuous but not contra-continuous.

Lemma 3 (Caldas et al. [7])**.** *Let A be a subset of (X,τ). Then the following properties hold:*

(1) *If A is preopen in (X,τ), then it is δ-preopen in (X,τ).*
(2) *A is δ-preopen in (X,τ) if and only if it is preopen in (X,τ_s).*
(3) *A is δ-preclosed in (X,τ) if and only if it is preclosed in (X,τ_s) .*

Since $\mathrm{Cl}(A) \subset \delta\mathrm{Cl}(A)$ for any subset A of X, therefore, every contra-precontinuous is contra-δ-precontinuous but not conversely as following example shows.

Example 4 ([5]). A contra-semi-continuous function need not be contra-precontinuous. Let $f:R \to R$ be the function $f(x) = [x]$, where $[x]$ is the Gaussian symbol. If V is a closed subset of the real line, its preimage $U = f^{-1}(V)$ is the union of the intervals of the form $[n,n+1]$, $n \in Z$; hence U is semi-open being union of semi-open sets. But f is not contra-precontinuous, because $f^{-1}(0.5,1.5) = [1,2)$ is not preclosed in R.

Example 5 ([5]). A contra-precontinuous function need not be contra-semi-continuous. Let $X = \{a,b\}$, $\tau = \{\varnothing,X\}$ and $\sigma = \{\varnothing,\{a\},X\}$. The identity function $f:(X,\tau) \to (Y,\sigma)$ is contra-precontinuous as only the trivial subsets of X are open in (X,τ). However, $f^{-1}(\{a\}) = \{a\}$ is not semi-closed in (X,τ); hence f is not contra-semi-continuous.

Example 6 ([6]). Let R be the set of real numbers, τ be the countable extension topology on R, i.e., the topology with subbase $\tau_1 \cup \tau_2$, where τ_1 is the Euclidean topology of R and τ_2 is the topology of countable complements of R, and σ be the discrete topology of R. Define a function $f:(R,\tau) \to (R,\sigma)$ as follows: $f(x) = 1$ if x is rational, and $f(x) = 2$ if x is irrational. Then f is contra δ-precontinuous but not contra-β-continuous, because $\{1\}$ is closed in (R,σ) and $f^{-1}(\{1\}) = Q$, where Q is the set of rationals, is not β-open in (R,τ).

Example 7 ([3]). Let $X = \{a,b,c\}$, $\tau = \{\varnothing,\{a\},\{b\},\{a,b\},X\}$ and $Y = \{p,q\}$, $\sigma = \{\varnothing,\{p\},Y\}$. Let $f:(X,\tau) \to (Y,\sigma)$ be defined by $f(a) = p$ and $f(b) = f(c) = q$. Then f is contra-β-continuous but not contra δ-precontinuous since $f^{-1}(\{q\}) = \{b,c\}$ is β-open but not δ-preopen.

Theorem 2. *If a function $f:X \to Y$ is contra δ-precontinuous and Y is regular, then f is δ-almost continuous.*

Proof. Let x be an arbitrary point of X and V an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $\mathrm{Cl}(W) \subset V$. Since f is contra δ-precontinuous, so by Theorem 1 there exists $U \in \delta PO(X,x)$ such that $f(U) \subset \mathrm{Cl}(W)$. Then $f(U) \subset \mathrm{Cl}(W) \subset V$. Hence, f is δ-almost continuous. □

The converse of Theorem 2 is not true. Example 1 shows that δ-almost continuity does not necessarily imply contra δ-precontinuity even if the range is regular.

Definition 4. A function $f: X \to Y$ is said to be:

(1) (δ,s)-preopen if $f(U) \in SO(Y)$ for every δ-preopen set of X.
(2) contra-$I(\delta,p)$-continuous if for each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in \delta PO(X,x)$ such that $\mathrm{Int}(f(U)) \subset F$.

Theorem 3. *If a function $f: X \to Y$ is contra-$I(\delta,p)$-continuous and (δ,s)-preopen, then f is contra δ-precontinuous.*

Proof. Suppose that $x \in X$ and $F \in C(Y, f(x))$. Since f is contra-$I(\delta,p)$-continuous, there exists $U \in \delta PO(X,x)$ such that $\mathrm{Int}(f(U)) \subset F$. By hypothesis f is (δ,s)-preopen, therefore $f(U) \in SO(Y)$ and $f(U) \subset \mathrm{Cl}(\mathrm{Int}(f(U)) \subset F$. This shows that f is contra δ-precontinuous. □

Definition 5. A space (X,τ) is said to be:

(1) locally (δ,p)-indiscrete if every δ-preopen set of X is closed in X.
(2) δp-space if every δ-preopen set of X is open in X.
(3) δS-space if and only if every δ-preopen subset of X is semi-open.

The following theorem follows immediately from Definition 5.

Theorem 4. *If a function $f: X \to Y$ is contra δ-precontinuous and X is a δS-space (resp., δp-space, locally (δ,p)-indiscrete), then f is contra-semi-continuous (resp., contra-continuous, continuous).*

Recall that a topological space is said to be:

(1) (δ,p)-T_2 ([16]) if for each pair of distinct points x and y in X there exist $U \in \delta PO(X,x)$ and $V \in \delta PO(X,y)$ such that $U \cap V = \varnothing$.
(2) Ultra Hausdorff [18] if for each pair of distinct points x and y in X there exist $U \in CO(X,x)$ and $V \in CO(X,y)$ such that $U \cap V = \varnothing$.

Theorem 5. *If X is a topological space and for each pair of distinct points x_1 and x_2 in X there exists a map f of X into a Urysohn topological space Y such that $f(x_1) \neq f(x_2)$ and f is contra δ-precontinuous at x_1 and x_2, then X is (δ,p)-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then by hypothesis, there is a Urysohn space Y and a function $f: X \to Y$, which satisfies the conditions of the theorem. Let $y_i = f(x_i)$ for $i = 1,2$. Then $y_1 \neq y_2$. Since Y is Urysohn, there exist open neighbourhoods U_{y_1} and U_{y_2} of y_1 and y_2 respectively in Y such that $\mathrm{Cl}(U_{y_1}) \cap \mathrm{Cl}(U_{y_2}) = \varnothing$. Since f is contra δ-precontinuous at x_i, there exists a δ-preopen neighbourhood W_{x_i} of x_i in X such that $f(W_{x_i}) \subset \mathrm{Cl}(U_{y_i})$ for $i = 1,2$. Hence we get $W_{x_1} \cap W_{x_2} = \varnothing$ because $\mathrm{Cl}(U_{y_1}) \cap \mathrm{Cl}(U_{y_2}) = \varnothing$. Then X is (δ, p)-T_2. □

Corollary 1. *If f is a contra δ-precontinuous injection of a topological space X into a Urysohn space Y, then X is (δ, p)-T_2.*

Proof. For each pair of distinct points x_1 and x_2 in X, f is a contra δ-precontinuous function of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ because f is injective. Hence by Theorem 5, X is (δ, p)-T_2 . □

Corollary 2. *If f is a contra δ-precontinuous injection of a topological space X into a Ultra Hausdorff space Y, then X is (δ, p)-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then since f is injective and Y is Ultra Hausdorff $f(x_1) \neq f(x_2)$, and there exist $V_1, V_2 \in CO(Y)$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \varnothing$. Then $x_i \in f^{-1}(V_i) \in \delta PO(X)$ for $i = 1,2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \varnothing$. Thus X is (δ, p)-T_2. □

Lemma 4 ([15])**.** *If A_i is a δ-preopen set in a topological space X_i for $i = 1,2,\ldots,n$, then $A_1 \times \cdots \times A_n$ is also δ-preopen in the product space $X_1 \times \cdots \times X_n$.*

Theorem 6. *Let $f_1 \colon X_1 \to Y$ and $f_2 \colon X_2 \to Y$ be two functions, where*

(1) *Y is a Urysohn space,*
(2) *f_1 and f_2 are contra δ-precontinuous.*

Then the set

$$\{(x_1, x_2) \mid f_1(x_1) = f_2(x_2)\}$$

is δ-preclosed in the product space $X_1 \times X_2$.

Proof. Let A denote the set $\{(x_1, x_2) \mid f_1(x_1) = f_2(x_2)\}$. In order to show that A is δ-preclosed, we show that $(X_1 \times X_2) \setminus A$ is δ-preopen. Let $(x_1, x_2) \notin A$. Then $f_1(x_1) \neq f_2(x_2)$. Since Y is Urysohn, there exist open V_1 and V_2 of $f_1(x_1)$ and $f_2(x_2)$ such that $C(V_1) \cap C(V_2) = \varnothing$. Since f_i $(i = 1,2)$ is contra δ-precontinuous, $f_i^{-1}(C(V_i))$ is a δ-preopen set containing x_i in X_i $(i = 1,2)$. Hence, by virtue of Lemma 4, $f_1^{-1}(C(V_1)) \times f_2^{-1}(C(V_2))$ is δ-preopen. Further $(x_1, x_2) \in f_1^{-1}C(V_1) \times f_2^{-1}C(V_2) \subset (X_1 \times X_2) \setminus A$. It follows that $(X_1 \times X_2) \setminus A$ is δ-preopen. Thus A is δ-preclosed in the product space $X_1 \times X_2$. □

Corollary 3. *If $f \colon X \to Y$ is contra δ-precontinuous and Y is a Urysohn space, then*

$$A = \{(x_1, x_2) \mid f(x_1) = f(x_2)\}$$

is δ-preclosed in the product space $X_1 \times X_2$.

Definition 6. A topological space X is said to be:

(1) (δ, p)-normal if each pair of non-empty disjoint closed sets can be separated by disjoint δ-preopen sets.
(2) Ultra normal [18] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 7. *If $f: X \to Y$ is a contra δ-precontinuous, closed injection and Y is ultra normal, then X is (δ, p)-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is ultra normal $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2, respectively. Hence $F_i \subset f^{-1}(V_i)$, $f^{-1}(V_i) \in \delta PO(X,x)$ for $i = 1,2$, and

$$f^{-1}(V_1) \cap f^{-1}(V_2) = \varnothing.$$

Thus, X is (δ, p)-normal. □

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