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SOME COMMON FIXED POINT THEOREMS FOR A CLASS OF FUZZY CONTRACTIVE MAPPINGS

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Abstract. The purpose of this paper is to state and prove a new lemma generalizing Lemma 3.1 of Arora and Sharma [1] and Proposition 3.2 of Lee and Cho [10]. Some common fixed point theorems for a type of fuzzy contractive mappings are also established. These theorems extend and generalize several previous results [3, 14, 21, 22].

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1. INTRODUCTION

Common fixed point theorems have been applied to diverse problems during the last few decades. These theorems provide techniques for solving a variety of applied problems in mathematical science and in dynamic programming (see, e. g., [4, 15, 16]). Extensions of the Banach contraction principle to multivalued mappings were initiated independently by Markin [11] and Nadler [13]. Therefore, results on fixed points of contractive type multivalued mappings have been carried out by many authors (see, for example, [2, 17, 21]).

The theory of fuzzy sets was investigated by Zadeh [24] in 1965. Some applications on results in this theory are discussed (see [9, 23]). In 1981, Heilpern [7] first introduced the concept of fuzzy contractive mappings and proved a fixed point theorem for these mappings in metric linear spaces. His result is a generalization of the fixed point theorem for point-to-set maps of Nadler [13]. Later, several fixed point theorems for types of fuzzy contractive mappings appeared (see, for instance, [1, 18–20]).

In this paper, we state and prove a new lemma generalizing Lemma 3.1 of Arora and Sharma [1] and Proposition 3.2 of Lee and Cho [10]. Two common fixed point theorems of a type of fuzzy contractive mappings are established. These theorems generalize and extend results in [3, 14, 21, 22]. Finally, we state a conclusion containing a brief of our results and future research.

2. Basic preliminaries

The definitions and terminologies for further discussions are taken from Heilpern [7]. Let (X,d) be a metric linear space. A *fuzzy set* in X is a function with domain X and values in $[0,1]$. If A is a fuzzy set and $x \in X$, then the function-value $A(x)$ is called the *grade of membership* of x in A. The collection of all fuzzy sets in X is denoted by $\mathfrak{F}(X)$.

Let $A \in \mathfrak{F}(X)$ and $\alpha \in [0,1]$. The *α-level set* of A, denoted by A_α, is defined by the formula

$$A_\alpha = \begin{cases} \{x : A(x) \geq \alpha\} & \text{if } \alpha \in (0,1], \\ \overline{\{x : A(x) > 0\}} & \text{if } \alpha = 0. \end{cases} \tag{2.1}$$

where $\overline{B}$ is the closure of a (nonfuzzy) set B.

Definition 1. A fuzzy set A in X is an *approximate quantity* if and only if its α-level set is a nonempty compact convex (nonfuzzy) subset of X for each $\alpha \in [0,1]$ and $\sup_{x \in X} A(x) = 1$.

The set of all approximate quantities, denoted $W(X)$, is a subcollection of $\mathfrak{F}(X)$.

Definition 2. Let $A, B \in W(X)$, $\alpha \in [0,1]$ and $CP(X)$ be the set of all nonempty compact subsets of X. Then one puts $p_\alpha(A,B) = \inf_{x \in A_\alpha, y \in B_\alpha} d(x,y)$, $\delta_\alpha(A,B) = \sup_{x \in A_\alpha, y \in B_\alpha} d(x,y)$, and $D_\alpha(A,B) = H(A_\alpha, B_\alpha)$, where H is the *Hausdorff metric* between two sets in the collection $CP(X)$.

We define the functions $p(A,B) = \sup_\alpha p_\alpha(A,B)$, $\delta(A,B) = \sup_\alpha \delta_\alpha(A,B)$, and $D(A,B) = \sup_\alpha D_\alpha(A,B)$.

Note that p_α is nondecreasing function of α.

Definition 3. Let $A, B \in W(X)$. Then A is said to be *more accurate* than B (or B includes A), denoted by $A \subset B$, if and only if $A(x) \leq B(x)$ for each $x \in X$.

The relation $\subset$ induces a partial ordering on $W(X)$.

Definition 4. Let X be an arbitrary set and Y be a metric linear space. F is said to be a *fuzzy mapping* if and only if F is a mapping from the set X into $W(Y)$, i. e., $F(x) \in W(Y)$ for each $x \in X$.

The following lemma and proposition are used in the sequel.

Lemma 1 ([12])**.** *Suppose that $\gamma : [0,\infty) \to [0,\infty)$ is a right continuous function such that $\gamma(t) < t$ for all $t > 0$. Then for every $t > 0$, $\lim_{n\to\infty} \gamma^n(t) = 0$, where γ^n is the nth iterate of γ, $n \in \mathbb{N} \cup \{0\}$.**

Proposition 1 ([13])**.** *If $A, B \in CP(X)$ and $a \in A$, then there exists $b \in B$ such that $d(a,b) \leq H(A,B)$.*

* $\mathbb{N}$ is the set of all positive integers

We consider the set Φ of all functions $\phi : [0,\infty)^5 \to [0,\infty)$ with the following properties:

(i) ϕ is nondecreasing with respect to each variable,
(ii) ϕ is right continuous with respect to each variable,
(iii) for each $t > 0$, $\Psi(t) = \max\{\phi(t,t,t,t,t), \phi(t,t,t,2t,0), \phi(t,t,t,0,2t)\} < t$.

3. Main results

Throughout this paper, let (X,d) be a metric space. We consider a subcollection of $\mathfrak{F}(X)$ denoted by $W^*(X)$. Each fuzzy set $A \in W^*(x)$, its α-level set is a nonempty compact (nonfuzzy) subset of X for each $\alpha \in [0,1]$. It is obvious that each element $A \in W(X)$ leads one to $A \in W^*(X)$ but the converse is not true. Now, we introduce the improvements of the lemmas in Heilpern [7] as follows.

Lemma 2. *If $\{x_0\} \subset A$ for each $A \in W^*(X)$ and $x_0 \in X$, then $p_\alpha(x_0,B) \leq D_\alpha(A,B)$ for each $B \in W^*(X)$.*

Lemma 3. *$p_\alpha(x,A) \leq d(x,y) + p_\alpha(y,A)$ for all $x,y \in X$ and $A \in W^*(X)$.*

Lemma 4. *Let $x \in X, A \in W^*(X)$ and $\{x\}$ be a fuzzy set with membership function equal to a characteristic function of the set $\{x\}$. Then $\{x\} \subset A$ if and only if $p_\alpha(x,A) = 0$ for each $\alpha \in [0,1]$.*

Proof. If $\{x\} \subset A$, then $x \in A_\alpha$ for each $\alpha \in [0,1]$. This implies that $p_\alpha(x,A) = \inf_{y\in A_\alpha} d(x,y) = 0$ for any $\alpha \in [0,1]$. Conversely, if $p_\alpha(x,A) = 0$, then we have $\inf_{y\in A_\alpha} d(x,y) = 0$. It follows that $x \in \overline{A_\alpha} = A_\alpha$ for an arbitrary $\alpha \in [0,1]$. Then $\{x\} \subset A$. □

Also, we state and prove a new lemma in the following way.

Lemma 5. *Let (X,d) be a complete metric space, $F : X \to W^*(X)$ be a fuzzy map and $x_0 \in X$. Then there exists $x_1 \in X$ such that $\{x_1\} \subset F(x_0)$.*

Proof. For $n \in \mathbb{N}$, $((F(x_0))_{n/(n+1)})$ is a decreasing sequence of nonempty compact subsets of X. Thus we have from Proposition 11.4 and Remark 11.5 of [25, pp. 495–496] that $\bigcap_{n=1}^{\infty}(F(x_0))_{n/(n+1)}$ is nonempty and compact.

Let $x_1 \in \bigcap_{n=1}^{\infty}(F(x_0))_{n/(n+1)}$. Then $\frac{n}{n+1} \leq (F(x_0))(x_1) \leq 1$. As $n \to \infty$, we get that $(F(x_0))(x_1) = 1$. This implies that $\{x_1\} \subset F(x_0)$. □

Remark 1. It is clear that Lemma 5 is a generalization of Lemma 3.1 of Arora and Sharma [1] and Proposition 3.2 of Lee and Cho [10].

Now, we are ready to prove our main theorems.

Theorem 1. *Let (X,d) be a complete metric space and F_1, F_2 be fuzzy mappings from X into $W^*(X)$. If there is a $\phi \in \Phi$ such that for all $x, y \in X$,*

$$D(F_1(x), F_2(y)) \leq \phi(d(x,y), p(x, F_1(x)), p(y, F_2(y)), p(x, F_2(y)), p(y, F_1(x))), \quad (3.1)$$

then there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$.

Proof. Let $x_0 \in X$. Then by Lemma 5, there exists $x_1 \in X$ such that $\{x_1\} \subset F_1(x_0)$. For $x_1 \in X$, the set $(F_2(x_1))_1$ is nonempty compact subset of X. Since $(F_1(x_0))_1$ and $(F_2(x_1))_1$ belong to $CP(X)$ and $x_1 \in (F_1(x_0))_1$, Proposition 1 asserts that there exists $x_2 \in (F_2(x_1))_1$ such that $d(x_1,x_2) \leq D_1(F_1(x_0), F_2(x_1))$. So, we have from Lemma 4 and the property (i) of ϕ that

$$\begin{aligned} d(x_1,x_2) &\leq D_1(F_1(x_0), F_2(x_1)) \leq D(F_1(x_0), F_2(x_1)) \\ &\leq \phi(d(x_0,x_1), \\ &\qquad p(x_0, F_1(x_0)), p(x_1, F_2(x_1)), p(x_0, F_2(x_1)), p(x_1, F_1(x_0))) \\ &\leq \phi(d(x_0,x_1), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1) + d(x_1,x_2), 0). \end{aligned}$$

If $d(x_1,x_2) > d(x_0,x_1)$, then

$$d(x_1,x_2) \leq \phi(d(x_1,x_2), d(x_1,x_2), d(x_1,x_2), 2d(x_1,x_2), 0) < d(x_1,x_2).$$

This contradiction demands that

$$d(x_1,x_2) \leq \phi(d(x_0,x_1), d(x_0,x_1), d(x_0,x_1), 2d(x_0,x_1), 0).$$

Similarly, one can deduce that

$$d(x_2,x_3) \leq \phi(d(x_1,x_2), d(x_1,x_2), d(x_1,x_2), 0, 2d(x_1,x_2)).$$

By induction, we have a sequence (x_n) of points in X such that, for all $n \in \mathbb{N} \cup \{0\}$,

$$\{x_{2n+1}\} \subset F_1(x_{2n}), \quad \{x_{2n+2}\} \subset F_2(x_{2n+1}).$$

It follows by induction that $d(x_n,x_{n+1}) \leq \Psi^n(d(x_0,x_1))$, where Ψ is defined in the property (iii) of ϕ. Then, Lemma 1 gives that $\lim_{n\to\infty} d(x_n,x_{n+1}) = 0$. Since

$$d(x_n,x_m) \leq d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + \ldots + d(x_{m-1},x_m),$$

then $\lim_{n,m\to\infty} d(x_n,x_m) = 0$. Therefore, (x_n) is a Cauchy sequence. Since X is a complete metric space, then there exists $z \in X$ such that $\lim_{n\to\infty} x_n = z$. Next, we show that $\{z\} \subset F_i(z), i = 1,2$. Now, we get from Lemma 2 and Lemma 3 that

$$\begin{aligned} p_\alpha(z, F_2(z)) &\leq d(z, x_{2n+1}) + p_\alpha(x_{2n+1}, F_2(z)) \\ &\leq d(z, x_{2n+1}) + D_\alpha(F_1(x_{2n}), F_2(z)), \end{aligned}$$

for each $\alpha \in [0,1]$. Taking the supremum on α in the last inequality, we obtain from the property (i) of ϕ that

$$\begin{aligned} p(z, F_2(z)) &\leq d(z, x_{2n+1}) + D(F_1(x_{2n}), F_2(z)) \\ &\leq d(z, x_{2n+1}) + \phi(d(x_{2n}, z), p(x_{2n}, F_1(x_{2n})), p(z, F_2(z)), \\ &\qquad p(x_{2n}, F_2(z)), p(z, F_1(x_{2n}))) \\ &\leq d(z, x_{2n+1}) + \phi(d(x_{2n}, z), d(x_{2n}, x_{2n+1}), p(z, F_2(z)), \\ &\qquad p(x_{2n}, F_2(z)), d(z, x_{2n+1})). \end{aligned}$$

As $n \to \infty$, we have from the properties (i), (ii) and (iii) of ϕ with $p(z, F_2(z)) \neq 0$ that

$$\begin{aligned} p(z, F_2(z)) &\leq \phi(0, 0, p(z, F_2(z)), p(z, F_2(z)), 0) \\ &\leq \phi(p(z, F_2(z)), p(z, F_2(z)), p(z, F_2(z)), p(z, F_2(z)), p(z, F_2(z))) \\ &< p(z, F_2(z)). \end{aligned}$$

This contradiction yields $p(z, F_2(z)) = 0$. We then get from Lemma 4 that $\{z\} \subset F_2(z)$. Similarly, one can show that $\{z\} \subset F_1(z)$. □

Example 1. Let $X = [0,1]$ endowed with the metric d defined by $d(x,y) = |x-y|$. It is clear that (X,d) is a complete metric space. Assume that $\phi(t_1, t_2, t_3, t_4, t_5) = \frac{3}{4}t_1$ for arbitrary $t_i \in [0,\infty)$, $i = \overline{1,5}$. It is obvious that $\Psi(t) < t$ for all $t > 0$. Let $F_1 = F_2 = F$. Define a fuzzy mapping F on X such that for all $x \in X$, $F(x)$ is the characteristic function for $\{\frac{3}{4}x\}$. For each $x, y \in X$,

$$\begin{aligned} D(F(x), F(y)) &= \frac{3}{4}d(x,y) \\ &= \phi(d(x,y), p(x, F(x)), p(y, F(y)), p(x, F(y)), p(y, F(x))). \end{aligned} \tag{3.2}$$

The characteristic function for $\{0\}$ is the fixed point of F.

As corollaries of Theorem 1, we get the following statements.

Corollary 1. *Let (X,d) be a complete metric space and F_1, F_2 be fuzzy mappings from X into $W^*(X)$ satisfying the following conditions: for any x, y in X,*

$$\begin{aligned} D(F_1(x), F_2(y)) \leq a_1 p(x, F_1(x)) + a_2 p(y, F_2(y)) + a_3 p(y, F_1(x)) \\ + a_4 p(y, F_1(x)) + a_5 d(x,y), \end{aligned} \tag{3.3}$$

where a_1, a_2, a_3, a_4, and a_5 are non-negative real numbers, $\sum_{j=1}^{5} a_i < 1$ and $a_1 = a_2$ or $a_3 = a_5$. Then there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$.

Proof. We consider the function $\phi : [0,\infty)^5 \to [0,\infty)$ defined by the formula

$$\phi(x_1, x_2, x_3, x_4, x_5) = a_1 x_2 + a_2 x_3 + a_3 x_5 + a_4 x_4 + a_5 x_1, \tag{3.4}$$

where $\sum_{i=1}^{i=5} a_i < 1$ such that $a_1 = a_2$ or $a_3 = a_4$. Since $\phi \in \Phi$, we have from Theorem 1 that there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$. □

The following corollary is a fuzzy version of the fixed point theorem of Singh and Whitfield [21] for multivalued mappings.

Corollary 2. *Let (X,d) be a complete metric space and F_1, F_2 be fuzzy mappings from X into $W^*(X)$. If there is a constant α, $0 \le \alpha < 1$, such that, for each $x, y \in X$,*

$$D(F_1(x), F_2(y)) \le \alpha \max\left\{ d(x,y), \frac{1}{2}[p(x,F_1(x)) + p(y,F_2(y))], \frac{1}{2}[p(x,F_2(y)) + p(y,F_1(x))] \right\}, \tag{3.5}$$

then there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$.

Proof. We consider the function $\phi : [0,\infty)^5 \to [0,\infty)$ defined by

$$\phi(x_1,x_2,x_3,x_4,x_5) = \alpha \max\left\{ x_1, \frac{1}{2}[x_2+x_3], \frac{1}{2}[x_4+x_5] \right\}. \tag{3.6}$$

Since $\phi \in \Phi$, we get from Theorem 1 that there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$. □

Remark 2. (1) If there is a $\phi \in \Phi$ such that, for each $x, y \in X$,

$$\delta(F_1(x), F_2(y)) \le \phi(d(x,y), p(x,F_1(x)), p(y,F_2(y)), p(x,F_2(y)), p(y,F_1(x))), \tag{3.7}$$

then the conclusion of Theorem 1 remains valid. This result is considered as a special case of Theorem 1 because $D(F_1(x),F_2(y)) \le \delta(F_1(x),F_2(y))$ [8, p. 414]. Moreover, this result generalizes Theorem 3.3 of Park and Jeong [14].

(2) Corollary 1 is [22, Theorem 3.1] without condition (a), where condition (a) reads as follows: "for each $x \in X$, there exists $\alpha(x) \in (0,1]$ such that $(F_1(x))_{\alpha(x)}$ and $(F_2(x))_{\alpha(x)}$ are nonempty closed bounded subsets of $\mathfrak{F}(X)$." Also, Corollary 1 generalizes [3, Theorem 3.1].

(3) Theorems 3.1 and 3.4 of Park and Jeong [14] are special cases of Theorem 1.

The following theorem generalizes Theorem 1 to a sequence of fuzzy contractive mappings.

Theorem 2. *Let $(F_n : n \in \mathbb{N} \cup \{0\})$ be a sequence of fuzzy mappings from a complete metric space (X,d) into $W^*(X)$. If there is a $\phi \in \Phi$ such that, for all $x, y \in X$,*

$$D(F_0(x), F_n(y)) \le \phi(d(x,y), p(x,F_0(x)), p(y,F_n(y)), p(x,F_n(y)), p(y,F_0(x))) \quad \forall (n \in \mathbb{N}), \tag{3.8}$$

then there exists a common fixed point of the family $(F_n : n \in \mathbb{N} \cup \{0\})$.

Proof. Putting $F_1 = F_0$ and $F_2 = F_n$ for all $n \in \mathbb{N}$ in Theorem 1. Then there exists a common fixed point of the family $(F_n : n \in \mathbb{N} \cup \{0\})$. □

Remark 3. If there is a $\phi \in \Phi$ such that, for all $x, y \in X$,

$$\delta(F_0(x), F_n(y)) \leq \phi(d(x,y), p(x, F_0(x)), p(y, F_n(y)), p(x, F_n(y)), p(y, F_0(x))) \qquad (\forall n \in \mathbb{N}), \quad (3.9)$$

then the conclusion of Theorem 2 remains valid. This result is considered as a special case of Theorem 2 for the same reason as in Remark 2 (1).

4. Conclusion

This paper presents an improvement of some results in [1, 7, 10]. Also, it presents two common fixed point theorems for a type of fuzzy contractive mappings. These theorems generalize and extend results in [3, 14, 22] and [21], respectively. A fixed point theorem for fuzzy contractive mappings is stated generalizing [1, Theorem 3.5]. Many applications of our main theorems are possible, e. g., for differential and integral equations. In view of the references [5, 6], some future research can be done, for example:

(1) I believe that our results can be hold for $FC(X)$, where $FC(X) = \{A \in \mathfrak{F}(X) : A_\alpha$ is a nonempty closed (nonfuzzy) subset of X for each $\alpha \in [0,1]\}$,

(2) it is also possible to generalize our results to quasi-metric spaces.

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